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Fiscal Stimulus in Expectations-Driven Liquidity Traps

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Fiscal Stimulus in Expectations-Driven Liquidity Traps*

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Abstract

I study liquidity traps in a model were agents have heterogeneous expectations and finite planning horizons. Backward-looking agents base their expectations on past observations, while forward-looking agents have fully rational expectations. Liquidity traps that are fully or partly driven by expectations can arise due to pessimism of backward-looking agents. Only when planning horizons are finite, these liquidity traps can be of longer duration without ending up in a deflationary spiral. I further find that fiscal stimulus in the form of an increase in government spending or a cut in consumption taxes can be very effective in mitigating the liquidity trap. A feedback mechanism of heterogeneous expectations causes fiscal multipliers to be the largest when the majority of agents is backward-looking but there also is a considerable fraction of agents that are forward-looking. Labor tax cuts are always deflationary and are not an effective tool in a liquidity trap.

Keywords: Bounded Rationality; Fiscal policy; Liquidity Trap; Heterogeneous Expectations

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1 Introduction

A large body of theoretical research has documented the state-dependence of fiscal multipliers. Christiano et al. (2009), Eggertsson (2011) and Woodford (2011) discuss why government spending multipliers are larger when the zero lower bound (ZLB) is binding than otherwise. Erceg and Lindé (2014) find that higher government spending can be used to shorten liquidity traps and even resolve them immediately if the stimulus is large enough. By contrast, lowering labor taxes is less effective, because this measure not just increases output, but also increases labor supply, implying lower wages. The resulting decrease in marginal costs for firms puts downward pressure on inflation, possibly increasing the severity and the duration of the liquidity trap.

So far, the majority of these studies have investigated the effectiveness of fiscal policy at the ZLB under the assumption of rational expectations and in the case of liquidity traps that are purely driven by fundamental shocks, i.e., shocks that reduce the natural rate of interest. An exception is Mertens and Ravn (2014), who investigate the occurrence of liquidity traps due to the coordination of expectations on a sunspot shock and find that, there, government spending increases are deflationary while labor tax cuts become inflationary.

However, a growing strain of the literature has shown the importance of bounded rationality for macroeconomic policy, especially at the ZLB (see a.o. Williams, 2006, Akerlof and Shiller, 2010, De Grauwe, 2012 and Gabaix, 2016). In this paper, I therefore study the effectiveness of fiscal stimulus with different fiscal instruments in liquidity traps that emerge due to boundedly rational and heterogeneous expectations. In particular, I compare three different types of liquidity traps: fundamentals-driven liquidity traps, where all agents have rational expectations and a persistent fundamental shock causes a binding zero lower bound; expectations-driven liquidity traps, that arise under heterogeneous expectations when a single non-persistent shock reduces output and inflation expectations of backward-looking agents; and mixed liquidity traps, where there both is a persistent fundamental shock and a fraction of backward-looking agents that amplify this shock with their expectations.

In line with e.g. Orphanides and Williams (2005, 2007), Milani (2007), Slobodyan and Wouters (2012) and Hommes and Zhu (2014), I find that the presence of backward-looking agents adds persistence. In both expectations-driven and mixed liquidity traps, this persistence amplification becomes so severe for larger fractions of backward-looking agents that the economy never recovers. Instead, the economy then ends up in a deflationary spiral. To tackle this issue, I propose a second intuitive layer of bounded rationality: finite planning horizons. The first main result of the paper is that relatively small planning horizons facilitate the existence of expectations-driven and mixed liquidity traps of considerable duration from which the economy eventually recovers. Such liquidity traps cannot arise under infinite planning horizons.

More specifically, I model bounded rationality as follows. Instead of being able to form expectations up to an infinite horizon as is usually assumed, agents in my model are relatively short-sighted and are able to plan ahead and form expectations only up to T periods into the future, as in Lustenhouwer and Mavromatis (2017) and Woodford (2018). Moreover, only a fraction of agents in the modeled economy form expectation in a forward-looking, rational manner. The other fraction of agents use a backward-looking rule of thumb, according to which all variables will mean-revert back to their steady state in the future. Such rule of thumb behavior, used by e.g. Branch and McGough (2009, 2010) and Gasteiger (2014, 2017), who label it adaptive expectations, is found to be consistent with expectations of human subjects in laboratory experiments (see e.g. Assenza et al. (2014) and Pfajfar and Zakelj, 2011) as well as with survey data (see e.g. Branch (2004, 2007)). Other works with similar heterogeneous expectations frameworks include Elton et al. (2017), Massaro (2013) and Deák et al. (2017). In terms of micro-foundations and aggregation of heterogeneous expectations the current paper is also related to these three papers. However, these papers assume that agents form their expectations over an infinite planning horizon, while my approach builds on the micro-foundations under finite planning horizons of Lustenhouwer and Mavromatis (2017) and Woodford (2018). To the best of my knowledge, this paper is the first to combine heterogeneity in expectations with a micro-founded framework of finite planning horizons.

Regarding the effects of government spending in expectations-driven liquidity traps, Evans et al. (2008), Evans and Honkapohja (2009) and Benhabib et al. (2014) find that under homogeneous adaptive learning, large enough government spending increases can always prevent deflationary spirals that would otherwise have arisen because of the ZLB. This result is confirmed by Hommes et al. (2015), who conduct a laboratory experiment, where, rather than making any assumptions on agent's expectations, they let expectations be formed by human subjects in the laboratory. Without fiscal intervention, deflationary spirals regularly occur in their experiment. However, in treatments where there is a fiscal switching rule and government spending is increased when inflation is below some threshold, deflationary spirals are always prevented. This is in sharp contrast with Mertens and Ravn (2014), who find that, contrary to widely held views, in their sunspot equilibria, increasing government spending at the zero lower bound is deflationary and is not very effective in mitigating a liquidity trap.

I confirm the results of the former papers and find that government spending increases are effective in ending liquidity traps and preventing deflationary spirals. Additionally, I show that the government spending multipliers become even larger under heterogeneous expectations. They are the largest when the fraction of backward-looking agents is relatively large but there also still is a significant fraction of forward-looking agents. Under this mixture of heterogeneous expectations, the following feedback mechanism arises. First, forward-looking agents expect the fiscal stimulus package to lead to higher inflation and output in the future, which leads them to increase their current consumption and prices. Next, backward-looking agents observe the resulting higher output and inflation and adjust their expectations and consumption and prices in subsequent periods. But this is already anticipated by forward-looking agents at the beginning of the liquidity trap, leading to an even higher initial increase in output and inflation and an even larger subsequent response of backward-looking agents. This feedback mechanism is present both in expectations-driven and in mixed liquidity traps.

I further find that labor taxes are deflationary in any type of liquidity trap and, depending on the fraction of backward-looking agents, even result in positive multipliers. Labor tax cuts are, therefore, not an effective stimulus tool in liquidity traps with bounded rationality and heterogeneous expectations. This is further indication that the reversal of traditional results found by Mertens and Ravn (2014) is not a general feature of liquidity traps driven by expectations, but depends crucially on the choice of modeling an expectations-driven liquidity trap as a sunspot equilibrium. When, instead expectations-driven liquidity traps are modeled using bounded rationality, fiscal instruments behave more in the way economic intuition would indicate.

Finally, I also consider cutting consumption taxes as a tool for fiscal stimulus. Unlike a cut in labor taxes, I find that a cut in consumption taxes is inflationary and can considerably reduce the duration of an expectations-driven liquidity trap. For fundamentals-driven liquidity traps, similar findings are presented in Eggertsson (2011), Coenen et al. (2012) and Correia et al. (2013). However, this paper is the first to study consumption tax cuts in expectations-driven liquidity traps and/or with bounded rationality. Under the benchmark calibration, I find multipliers for consumption taxes to be somewhat smaller (in absolute value) than those of government spending. However, the size of consumption tax multipliers is affected in the same way by the feedback-mechanism described above, and multipliers become considerably bigger than 1 (in absolute value) for large, but not too large fractions of backward-looking agents.

Apart from the papers mentioned earlier , related works with finite planning horizons include Branch et al. (2013) and Evans et al. (2019). Heterogeneous expectations in new Keynesian models have further been studied in amongst others Kurz et al. (2013), Pecora and Spelta (2017) and De Grauwe and Ji (2019). To the best of my knowledge, no paper with heterogeneous expectations or finite planning horizons studies fiscal policy under the zero lower bound, though.

The remainder of the paper is organized as follows. In Section 2, the model and expectation formation processes are outlined. In Section 3, I present how different types of liquidity traps can occur under heterogeneous expectations and show that liquidity traps of longer duration only arise under finite planning horizons. In Section 4, the effectiveness of different fiscal stimulus packages is investigated. Finally, Section 5 concludes.

2 Model

The model is made up by a continuum of households $i \in [0,1]$, a continuum of firms $j \in [0,1]$ and a monetary and fiscal authority. Moreover, there are two types of households and firms. A fraction α of households and firms forms expectations in a backward-looking manner and a fraction $1-\alpha$ forms expectations in a forward-looking manner. The expectations of these two types of households and firms will be specified in Section 2.4. Section 2.1 presents the optimization problem and first order conditions of households, Section 2.2 that of firms, and the government sector, monetary policy rule and market clearing are presented in Section 2.3. In Appendix B the model is log-linearized and aggregated.

2.1 Households

Households want to maximize their discounted utility over their planning horizon (T periods), and they also value the state they expect to end up in at the end of these T periods (their state in period T+1). They are not able to rationally induce (by solving the model forward), how exactly they should value their state in period T+1. Instead, households use a rule of thumb to evaluate the value of their state (their wealth). As in Lustenhouwer and Mavromatis (2017) and Woodford (2018), their objective function therefore exists of a sum of utilities, $U(\cdot)$, out of consumption and leisure for the periods within their horizon, as well as an extra term with a function $V(\cdot)$ that is increasing in end of horizon wealth:

$$\max_{C^i,H^i,B^i} \tilde{E}_t^i \left[\sum_{s=t}^{t+T} \beta^{s-t} \xi_s U(C_s^i, H_s^i) + \beta^{T+1} V\left(\frac{B_{t+T+1}^i}{P_{t+T}} \right) \right], \tag{1}$$

subject to

$$(1+\tau_\tau^c)P_\tau C_\tau^i + \frac{B_{\tau+1}^i}{1+i_\tau} \leq (1-\tau_\tau^l)W_\tau H_\tau^i + B_\tau^i + P_\tau \Xi_\tau - P_\tau LS_\tau, \qquad \tau = t, t+1, ...t+T. \tag{2}$$

Here B_t^i are nominal bond holdings from household i at the beginning of period t; C_τ^i and H_τ^i are the household's consumption and labor; Ξ_t are real profits from firms which are equally distributed among households; τ_τ^c and τ_τ^l are respectively the consumption tax and labor tax rates; LS_τ denotes lump sum taxes; i_τ is the nominal interest rate; P_τ is the price level; and W_τ is the nominal wage rate. Finally β is the household's discount factor, while ξ_τ is an exogenous preference shock.

Dividing the budget constraint by P_τ gives

$$(1+\tau_\tau^c)C_\tau^i + \frac{B_{\tau+1}^i}{(1+i_\tau)P_\tau} \leq (1-\tau_\tau^l)w_\tau H_\tau^i + \frac{B_\tau^i}{P_\tau} + \Xi_\tau - LS_\tau, \qquad \tau = t, t+1, ...t+T. \tag{3}$$

It is assumed that households have CRRA preferences for consumption and labor, so that

$$U(C_s^i, H_s^i) = \frac{(C_s^i)^{1-\sigma}}{1-\sigma} - \frac{(H_s^i)^{1+\eta}}{1+\eta}. \tag{4}$$

Moreover, the functional form of $V(.)$ is given by

$$V(x) = \frac{1}{1-\beta}\left[\frac{1}{1-\sigma}\left(\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{1+\bar{\tau}^c}\frac{x}{\bar{\Pi}}\right)^{1-\sigma}\right], \tag{5}$$

with $\Lambda = (1-\bar{\tau}^l)\bar{w}\bar{H} + \bar{\Xi}$ equal to steady state net income.

Equation (5) is (dropping terms independent of x) the continuation value that solves the Bellman equation

$$V(x) = \max_c\{U(C, \bar{H}) + \beta V(x')\}, \quad s.t. \quad x' = \frac{\bar{\Pi}}{\beta}\left[(1-\bar{\tau}^l)\bar{w}\bar{H} + \frac{x}{\bar{\Pi}} + \bar{\Xi} - \bar{L}S - (1+\bar{\tau}^c)C\right]. \tag{6}$$

Similar to Woodford (2018), this optimization problem gives the optimal intertemporal consumption decision of households assuming that taxes, wages, hours worked, inflation, interest rates and profits are all *in steady state*. That is, the only variables that are allowed to vary under this optimization problem are consumption (C) and debt ($x = \frac{B^i_{t+T+1}}{P_{t+T}}$).

Under this way of deriving (5), agents are not sophisticated enough to plan how their hours worked, wages and aggregate variables like inflation, interest rates and profits would change after their horizon if they would vary their consumption plan after their horizon.

However, using this Value function in Equation (1), households make fully optimal decisions in steady state. Moreover, $V(x)$ is increasing in x. Therefore, agents realize that holding more bonds at the end of their horizon will result in more utility. The value function hence captures partly how future utility depends on end-of-horizon wealth, but in a boundedly rational manner that only approximates the true value function.

The first order conditions of the maximization problem (1) subject to (3) are

$$\xi_\tau (C^i_\tau)^{-\sigma} = \lambda^i_\tau (1+\tau^c_\tau), \qquad \tau = t, t+1, ...t+T, \tag{7}$$

$$\xi_\tau (H^i_\tau)^{\eta} = \lambda^i_\tau (1-\tau^l_\tau) w_\tau, \qquad \tau = t, t+1, ...t+T, \tag{8}$$

$$\lambda^i_\tau = \beta \tilde{E}^i_t \frac{(1+i_\tau)\lambda^i_{\tau+1}}{\Pi_{\tau+1}}, \qquad \tau = t, t+1, ...t+T-1, \tag{9}$$

$$\lambda^i_{t+T} = \beta(1+i_{t+T}) \frac{1}{\bar{\Pi}(1+\bar{\tau}^c)} \left(\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{(1+\bar{\tau}^c)\bar{\Pi}} \frac{B^i_{t+T+1}}{P_{t+T}} \right)^{-\sigma}, \tag{10}$$

Next, we define a measure of real bond holdings, scaled by steady state output: $b_t = \frac{B_t}{P_{t-1}\bar{Y}}$. Substituting for this expression in (10) and (3) gives

$$\lambda^i_{t+T} = \beta(1+i_{t+T}) \frac{1}{\bar{\Pi}(1+\bar{\tau}^c)} \left(\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{(1+\bar{\tau}^c)} \frac{\bar{Y} b^i_{t+T+1}}{\bar{\Pi}} \right)^{-\sigma}, \tag{11}$$

and

$$(1+\tau^c_\tau) C^i_\tau + \bar{Y} \frac{b^i_{\tau+1}}{1+i_\tau} \leq (1-\tau^l_t) w_\tau H^i_\tau + \frac{\bar{Y} b^i_\tau}{\Pi_\tau} + \Xi_\tau - LS_\tau, \qquad \tau = t, t+1, ...t+T. \tag{12}$$

2.2 Firms

There is a continuum of firms producing the final differentiated goods. Each firm has a linear technology with labor as its only input,

$$Y_t(j) = H_t(j). \tag{13}$$

There is monopolistic competition and it is assumed that in each period a fraction $(1-\omega)$ of firms can change their price, as in Calvo (1983).

Each firm is run by a household and follows the same heuristic for prediction of future variables as that household in each period. Moreover, firms are also short sighted. That is, they will form expectations about their marginal costs and the demand for their product for T periods ahead only. However, as in the case of the household problem, firms also care about their state at the end of the horizon, and consider the possibility that they might then still be stuck with the price that they set now. The problem of firm j that can reset its price is then to maximize the discounted sum of its expected future profits within its horizon plus its perceived value of its state at the end of the horizon. In utility terms, and using the demand for good j, this can be written as

$$\tilde{E}_t^j \left(\sum_{s=0}^{T} \omega^s \beta^s \lambda_{t+s}^j \left[\left(\frac{p_t(j)}{P_{t+s}} \right)^{1-\theta} Y_{t+s} - mc_{t+s} \left(\frac{p_t(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s} \right] + \omega^{T+1} \beta^{T+1} \tilde{V}\left(\frac{p_t(j)}{P_{t+T}}\right) \right), \tag{14}$$

where λ_t^j is the Lagrange multiplier of the utility optimization problem of the household (j) that runs firm j.

As in Woodford (2018), $\tilde{V}(\cdot)$ describes the continuation value of real profits in utility terms as a function of the relative price. As in case of the household, this value function is obtained from the assumption that all variables other than the relative price of the firm (such as output, wages and the aggregate price level) are in steady state. This value function therefore satisfies

$$\tilde{V}(r) = \bar{\lambda} \left(\left(\frac{r}{\bar{\Pi}} \right)^{1-\theta} \bar{Y} - \left(\frac{r}{\bar{\Pi}} \right)^{-\theta} \bar{Y} \bar{mc} \right) + \omega\beta\tilde{V}\left(\frac{r}{\bar{\Pi}}\right) + (1-\omega)\beta\tilde{V}^{opt}, \tag{15}$$

where $\tilde{V}^{opt}$ is next period's value for a firm that can re-optimize next period. Since $\tilde{V}^{opt}$ does not influence the current decision problem of the firm (since it is independent of r), I ignore it and let the functional form of $\tilde{V}(r)$ be

$$\tilde{V}(r) = \frac{1}{1-\omega\beta\bar{\Pi}^{\theta-1}} \bar{\lambda} \left(\frac{r}{\bar{\Pi}} \right)^{1-\theta} \bar{Y} - \frac{1}{1-\omega\beta\bar{\Pi}^{\theta}} \bar{\lambda} \left(\frac{r}{\bar{\Pi}} \right)^{-\theta} \bar{Y} \bar{mc}. \tag{16}$$

The first order condition for maximizing (14) with respect to $p_t(j)$ then is

$$\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\lambda_{t+s}^j}{P_{t+s}} Y_{t+s} \left[(1-\theta) \left(\frac{p_t^*(j)}{P_{t+s}} \right)^{-\theta} + \theta mc_{t+s} \left(\frac{p_t^*(j)}{P_{t+s}} \right)^{-1-\theta} \right] \tag{17}$$
$$+(\omega\beta)^{T+1} \frac{\bar{\lambda}}{\bar{\Pi} P_{t+T}} \bar{Y} \left[\frac{1-\theta}{1-\omega\beta\bar{\Pi}^{\theta-1}} \left(\frac{p_t^*(j)}{\bar{\Pi} P_{t+T}} \right)^{-\theta} + \frac{\theta \bar{mc}}{1-\omega\beta\bar{\Pi}^{\theta}} \left(\frac{p_t^*(j)}{\bar{\Pi} P_{t+T}} \right)^{-1-\theta} \right] = 0,$$

where $p_t^*(j)$ is the optimal price for firm j if it can re-optimize in period t.

Next, turn to the evolution of the aggregate price level. I assume that the set of firms that can change their price in a period is chosen independently of the type of the household running the firm, so that the distribution of expectations of firms that can change their price is identical to the distribution of expectations of all firms. Since decisions of firms only differ in so far as their expectations differ, it follows that the aggregate price level evolves as

$$P_t = [\omega P_{t-1}^{1-\theta} + (1-\omega) \int_0^1 p_t^*(j)^{1-\theta} \mathrm{d}j]^{\frac{1}{1-\theta}}. \tag{18}$$

2.3 Completing the model

The government issues bonds and levies labor taxes (τ_t^l), consumption taxes (τ_t^c) and lump sum taxes (LS_t) to finance its (wasteful) spending (G_t). Its budget constraint is given by

$$\frac{B_{t+1}}{1+i_t} = P_t G_t - \tau_t^l W_t H_t - \tau_t^c P_t C_t - P_t LS_t + B_t, \tag{19}$$

with $H_t = \int H_t^i di$ and $B_t = \int B_t^i di$ aggregate labor and aggregate bond holdings respectively. Dividing by $\bar{Y} P_t$ gives

$$\frac{b_{t+1}}{1+i_t} = g_t - \tau_t^l w_t \frac{H_t}{\bar{Y}} + \tau_t^c \frac{C_t}{\bar{Y}} - \frac{LS_t}{\bar{Y}} + \frac{b_t}{\Pi_t}, \tag{20}$$

where $b_t = \frac{B_t}{P_{t-1}\bar{Y}}$ and $g_t = \frac{G_t}{\bar{Y}}$ are the ratios of debt to steady state GDP and government expenditure to steady state GDP, respectively.

Market clearing is given by

$$Y_t = C_t + G_t = C_t + \bar{Y} g_t. \tag{21}$$

g_t, τ_t^l and τ_t^c can be set in a discretionary manner by the government to counteract liquidity traps. Only lump sum taxes, LS_t, adjust to stabilize debt,

$$LS_t = \bar{LS} \left(\frac{b_t}{\bar{b}} \right)^{\gamma_{LS}}. \tag{22}$$

The monetary policy rule is given by

$$1 + i_t = \max \left(1, (1 + \bar{i}) \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_1} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_2} \right). \tag{23}$$

2.4 Expectations

There are two types of agents in the economy: forward-looking agents and backward-looking agents. Forward-looking agents are assumed to form fully rational expectations.

Backward-looking agents, on the other hand, consider the last observation of all variables, and consider this observation to be most informative about the current state of the economy, and its future evolution. They, however, do not expect the economy to stay in its current state forever, but instead expect mean-reversion to the target steady state in the future. Their expectations about government spending s periods from now are therefore given by

$$E_t^b \tilde{g}_{t+s} = \rho^{s+1} \tilde{g}_{t-1} \tag{24}$$

Branch and McGough (2010) and Gasteiger (2014) and others refer to these expectations as adaptive expectations. Expectations about output, inflation, debt, the nominal interest rate, taxes and price dispersion are formed analogously.

However, as will be discussed below, I allow for a shock to output and inflation expectations of backward-looking agents, $\hat{\epsilon}_t$. This shock, lets them form expectations as if they observed past output that was $\hat{\epsilon}_t$ higher than it actually was. For simplicity, it is assumed that these shocks affect all individual backward-looking agents equally. The output and inflation expectations of all backward-looking agents then become

$$E_t^b \hat{Y}_{t+s} = \rho^{s+1}(\hat{Y}_{t-1} + \hat{\epsilon}_t) \tag{25}$$

$$E_t^b \hat{\pi}_{t+s} = \rho^{s+1}(\hat{\pi}_{t-1} + \hat{\epsilon}_t) \tag{26}$$

3 Liquidity traps

In this section, I show that in the above model with forward-looking and backward-looking agents, different types of liquidity traps may arise. In particular, there can be liquidity traps purely driven by fundamentals, purely driven by expectations, or partly driven by fundamentals and partly by expectations. I will refer to these three types as 'fundamentals-driven liquidity traps', 'expectations-driven liquidity traps' and 'mixed liquidity traps'.

Purely fundamentals-driven liquidity traps can only arise when all agents in the economy are forward-looking. A liquidity trap then arises when the economy is hit by a persistent shock to the fundamentals of the economy. The shock chosen to illustrate this case is a persistent negative preference shock that creates a desire to save (ξ_t in Equation (1)). Similar shocks are used to model a liquidity trap by e.g. Eggertsson (2011) and Mertens and Ravn (2014).

In contrast, purely expectations-driven liquidity traps can arise only if there is a considerable fraction of backward-looking agents in the economy, and if the expectations of these agents are hit by a non-persistent negative shock. In order to highlight the role of expectations and keep the analysis as general as possible, I initiate such a liquidity trap by a shock directly to both output and inflation expectations ($\hat{\epsilon}_t$ in Equations (25) and (26)). The initial fall in expectations could then be though of as having been caused by a single, non-persistent shock to fundamentals, but one could also imagine that the fall in expectations was caused by something outside the model such as a global panic, or a financial crash.

The intuition for an expectations-driven liquidity trap of multiple periods to arise in the behavioral model, even after a non-persistent shock to expectations, is the following. Because of low output and inflation expectations, agents reduce consumption and prices, so that output and inflation fall. This reinforces the low expectations of backward-looking agents, and the liquidity trap continues.

Finally, mixed liquidity traps can arise if the economy is a hit by a persistent fundamental shock and part of the agents in the economy are backward-looking. In this case, the persistent fundamental shock causes a liquidity trap of multiple periods to arise, but the liquidity tap is made worse and lasts longer because of pessimistic expectations of backward-looking agents.

Below, I start with infinite planning horizons, and compare the standard fundamentals-driven liquidity traps with mixed and expectations-driven liquidity traps in Section 3.2. I then show in Section 3.3 that the deflationary spirals that often arise for the latter two cases largely disappear when agents have a short rather than an infinite planning horizon. Instead, liquidity traps of longer duration from which the economy can eventually recover now arise. The mechanisms behind such liquidity traps are illustrated in Section 3.4. First, the parameterization is discussed in Section 3.1.

3.1 Parameterization

In the model, one period corresponds to one quarter. I set the discount factor to $\beta = 0.99$, the coefficient of relative risk aversion to $\sigma = 1.5$, the inverse of the Frisch elasticity of labor supply to $\eta = 2$, the elasticity of substitution to $\theta = 6$ and the Calvo parameter to $\omega = 0.75$. These values are relatively standard in the literature.

Steady state fiscal variables are chosen more or less in line with US historical averages as follows: steady state government spending as a share of GDP is set to $\bar{g} = \bar{G}/\bar{Y} = 0.3$; the steady state labor and consumption tax rate are set to respectively $\bar{\tau}^l = 0.2$ and $\bar{\tau}^c = 0.08$, and steady state lump sum taxes are set to $\bar{LS} = 0.08$. The inflation target is set to 2%. Other monetary and fiscal policy parameters are set to $\phi_1 = 1.5$ and $\phi_2 = 0.157$ (implying a response to output of around 0.6 when annual data are used) and $\gamma_{LS} = 1$.

I further set the mean reversion in the expectations of backward-looking agents to 0.8. With that calibration, backward-looking agents expect the deviation of variables from steady state to have reduced to one tenth of the current deviation after approximately 10 quarters. The autocorrelation parameter in the preference shock is also set to 0.8.

3.2 Durations of liquidity traps

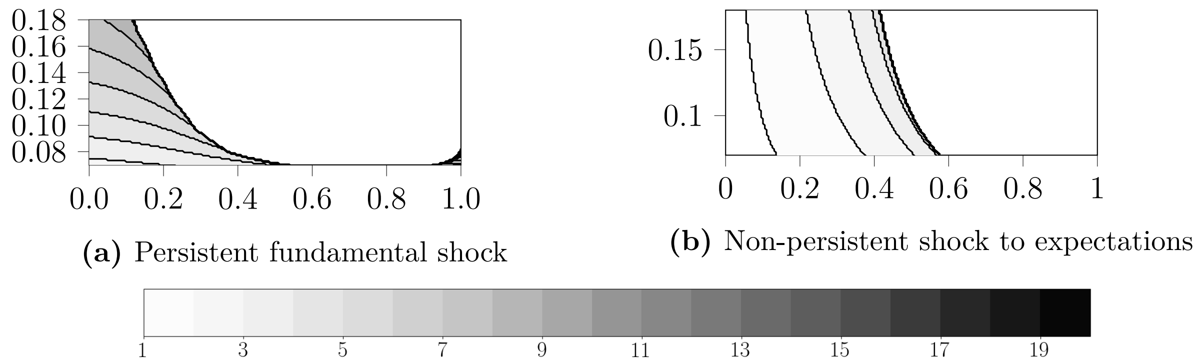
First, consider the case where agents have an infinite planning horizon. This allows us to study, in isolation, the effect of heterogeneous expectations on the duration of liquidity traps. I do so for two cases: persistent negative preference shocks and non-persistent shocks to expectations.

Panel (a) of Figure 1 corresponds to the case of a persistent negative preference shock. Let's start with only the most left part of the panel. Here, there are no backward-looking agents. That is, all agents have fully rational expectations as well as a (standard) infinite planning horizon. Liquidity traps that arise here are **fundamentals-driven** liquidity traps. The duration of these liquidity trap will, of course, depend on the size of the persistent shock that hits the economy. The size of this shock is varied along the y-axis of the Figure (where its absolute value is displayed). Darker shades of gray inside the figure indicate longer liquidity traps. It can, therefore, be concluded that, for the shock sizes considered here, the duration of the fundamentals-driven liquidity trap varies from 2 to 7 periods.[1]

As we move to the right in Panel (a) the fraction of backward-looking agents is gradually increased form 0 to 1 (along the x-axis). The interior of the panel therefore corresponds to **mixed** liquidity traps. For most shock sizes, moving to the right in the panel does not results in longer liquidity traps from which the economy eventually recovers (darker shades of gray). Instead, as the fraction of backward-looking agents is increased further, a deflationary spiral arises from which the economy never recovers. This is indicated by the fully white area in the top right part of the panel.

Next, consider panel (b) of Figure 1. Here, there is no shock to fundamentals. Instead, there is a single, non-persistent shock to the output and inflation expectations of backward-looking agents. As a consequence, no liquidity trap arises when the fraction of backward-looking agents is small, even for very large shock sizes. For larger fractions of backward-looking agents, liquidity traps do arise. These are **expectations-driven** liquidity traps. As was the case for mixed liquidity traps, above, larger fractions of backward-looking agents quickly lead to deflationary spirals, and intermediate cases where liquidity traps last for more than three or four periods but do not end up in an deflationary spiral do not occur.

[1] The duration of the liquidity trap for different shock sizes and different fractions of backward-looking agents are each time calculated with a single simulation where the shock that initiates the liquidity trap is the only shock innovation. These simulations therefore are deterministic. This can be done in this manner since the only nonlinearity in the log-linearized model is the zero lower bound. Therefore, the model is monotonic in further shock innovations. Obtaining durations of liquidity traps after a single (possibly persistent) shock hence gives the same result as simulating the model many times with different further shock innovations and using medians to obtain the duration of the liquidity trap. The latter will, however, be done later on to calculate the 0.05 and 0.95 quantiles in illustrative simulations.



(a) Persistent fundamental shock

(b) Non-persistent shock to expectations

Figure 1: Durations of liquidity traps for different fractions of backward-looking agents (x-axis) and different shock sizes sizes (y-axis) when agents have infinite planning horizons. Panel (a) captures both fundamentals-driven and mixed liquidity traps, while Panel (b) captures expectations-driven liquidity traps. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas in the top right indicate deflationary spirals.

3.3 Deflationary spirals and planning horizons

Above, we found that larger fractions of backward-looking agents in mixed or expectations-driven liquidity traps do not lead to longer liquidity traps from which the economy eventually recovers but, instead, to deflationary spirals. The intuition for such a deflationary spiral to arise is that backward-looking agents expect low inflation, low output and high real interest rates for many future periods and hence reduce prices and consumption with considerable magnitude. When there are enough backward-looking agents in the economy, the resulting drop in inflation and output is enough to make agents even more pessimistic in the next period, causing inflation and output to keep falling further and further.

This mechanism, however, crucially depends on agents first being able to form concrete expectations also about periods that are relatively far in the future and then being able to include all future periods in their optimization problem. Only under these two assumption will their current decisions be affected by what they believe will happen in all future periods.

Results can become quite different if agents have finite planning horizons due to limited cognitive ability. If agents are not able to form detailed expectations about periods that are more the T periods in the future and/or are not able to let these expectations enter their current optimization problem in a sophisticated manner, then current consumption and pricing decisions will not be impacted so much by pessimistic expectations about the future. As a consequence, output and inflation will fall less in the subsequent period, limiting the continuation of severe pessimism and possibly averting a deflationary spiral.

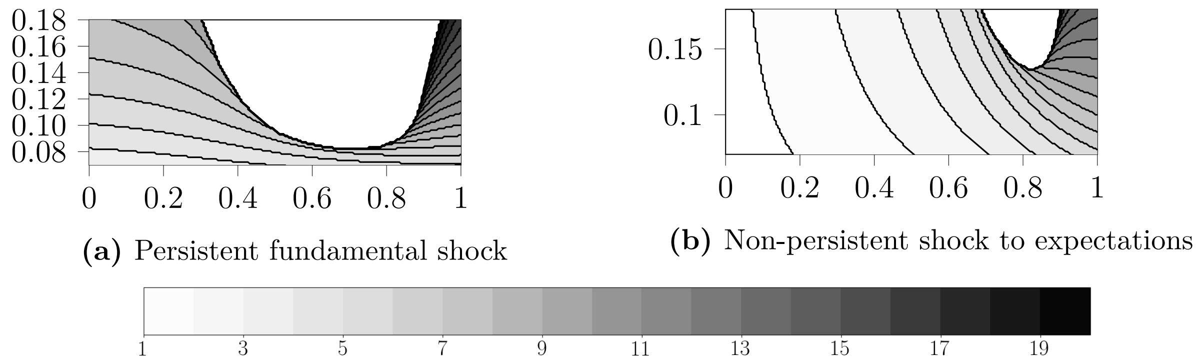
Figure 2 shows that this is indeed the case for $T = 4$.[2] The figure reproduces Figure 1 but now with agents having a finite planning horizon of 4 periods. It can immediately be seen in the figure that the white areas where deflationary spirals occur have become smaller. Moreover, the largest effect occurs for very large fractions of backward-looking agents (most right parts of both panels). Instead of deflationary spirals, long lasting liquidity traps (dark shades of gray) arise here for larger shocks.

That is, when agents have finite (small) planning horizons and there is a large fraction of backward-looking agents in the economy, long lasting expectation driven and mixed liquidity traps can arise from which the economy eventually recovers. This in contrast with the case of infinite planning horizons for which large fractions of backward-looking in combination with larger shocks always lead to deflationary spirals. This is the first main result of the paper.

3.4 Illustration of expectations-driven liquidity traps

To get further intuition in long lasting liquidity traps from which the economy eventually recovers, Figure 3 presents impulse response functions of such a liquidity trap. Here, the

[2] Four quarters might seem like a relatively small number, but survey evidence suggests that many households might indeed have such short planning horizons. For example, Fulda and Lersch (2018) present the results of the Household, Income and Labour Dynamics in Australia (HILDA) Survey, where the mean response to the financial planning horizon is between "The next few months" and "The next year". Hong and Hanna (2014) report a somewhat longer median response of "the next few years" in the Survey of Consumer Finances (SCF). However, also here, over 20% gave "the next few months" as a response.



(a) Persistent fundamental shock

(b) Non-persistent shock to expectations

Figure 2: Durations of liquidity traps for different fractions of backward-looking agents (x-axis) and different shock sizes sizes (y-axis) when agents have a finite planning horizon of $T = 4$ periods. Panel (a) captures both fundamentals-driven and mixed liquidity traps, while Panel (b) captures expectations-driven liquidity traps. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas in the top indicate deflationary spirals.

fraction of backward-looking agents is set to 0.8 and the size of the negative shock to expectations to 0.11 which is equivalent to a reduction of last periods output and inflation of 11% (see equations (25) and (26)).

Due to the nonlinearity of the zero lower bound, the exact impulse responses depend on the realizations of further shock innovations. To give an indication of the paths of endogenous variables that might arise, I generate 1000 sequences of different random draws of the preference shock innovations along the simulation path.[3] For each of the 1000 random sequences, I simulate the model once *with* the negative shock to expectations and once without, in order to then subtract the later time series from the former. The solid curves in Figure 3 correspond to the median of the resulting 1000 impulse responses and the dotted curves depict the 0.05 and 0.95 quantiles. Note that the median responses are equal to the responses that are obtained in a deterministic simulation without preference shocks (see footnote 1).

The bottom two panels show the one-period-ahead expectations of backward-looking agents in purple. It can be seen that, due to the shock, one-period-ahead expectations of

[3]The standard deviation of these shock innovations is set to 0.5%.

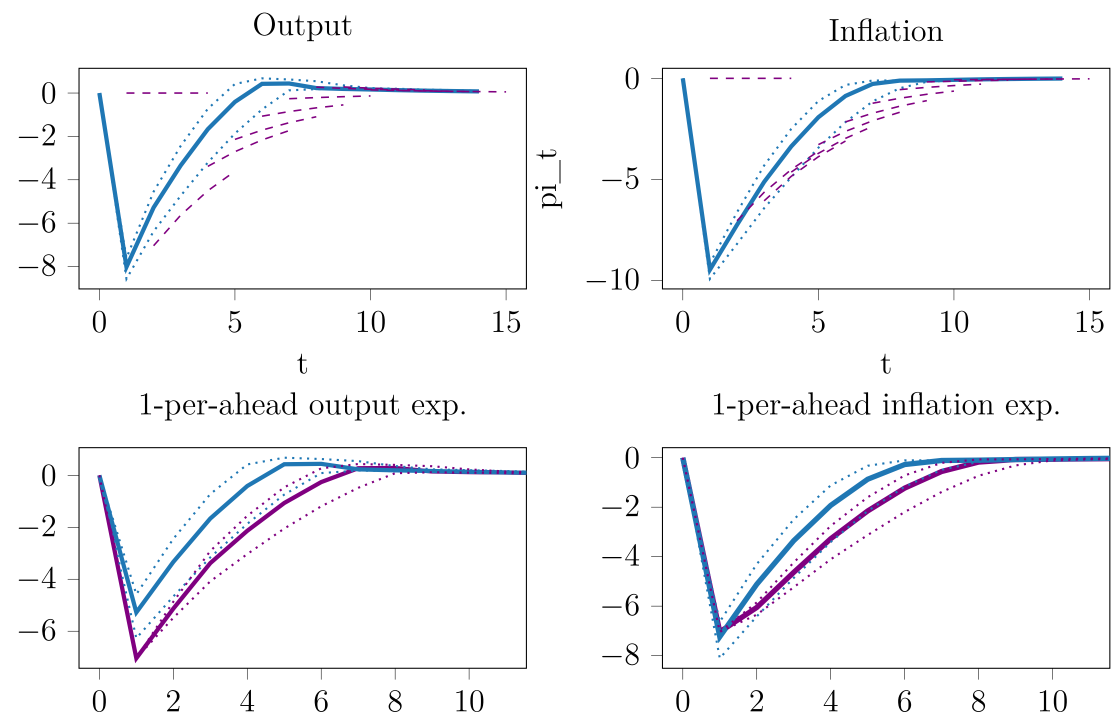


Figure 3: Expectations-driven liquidity trap for $T = 4$ and $\hat{\epsilon}_1 = -0.11$ and 80% backward-looking agents. The blue solid curves in the top panels depcits median impulse responses of actual output and inflation. The dashed purple curves represent time paths that are expected by backward-looking agents. The solid purple and solid blue curves in the bottom panels depict median one-period-ahead expectations of respectively backward-looking and foreward-looking agents. Dotted curves, in all panels, depict 0.05 and 0.95 quantiles.

backward-looking agents suddenly become low in period 1. As a consequence, backward-looking agents want to reduce their consumption and prices. Since the zero lower bound becomes binding, the interest rate is not reduced enough to stabilize output and inflation and both fall considerably in period 1 (top two panels). In period 2, this causes backward-looking agents to still have low output and inflation expectations, even though the shock to their expectations is over.

Forward-looking agents anticipate this lower path of actual output and inflation and also reduce their expectations in period 1. This is illustrated in the blue curves in the bottom panels that show the one-period-ahead expectations of forward-looking agents. Comparing the blue and purple curves, it can be seen that inflation expectations of both agent types are very similar in period 1. This indicates that the pessimistic inflation expectations of backward-looking agents become self-fulfilling.

In the end, output and inflation slowly recover, and the zero lower bound remains binding for 5 periods. If the fraction of backward-looking agents were larger, their expectations would become even more self-fulfilling implying slower recovery and a longer liquidity trap.

Finally, the dashed purple curves in the top panels correspond to the path of inflation that backward-looking agents expect at different points in time. Since agents have a planning horizon of $T = 4$, each purple curve spans four future periods. If agents would instead have had infinite planning horizons, they would form pessimistic expectations (and base their current decision on these expectations) also for periods further in the future. This would lead them to reduce consumption and prices considerably more in period 1. Their expectations in period 2 would then become even lower than in period 1, and a deflationary spiral would be unavoidable.

4 Fiscal stimulus in a liquidity trap

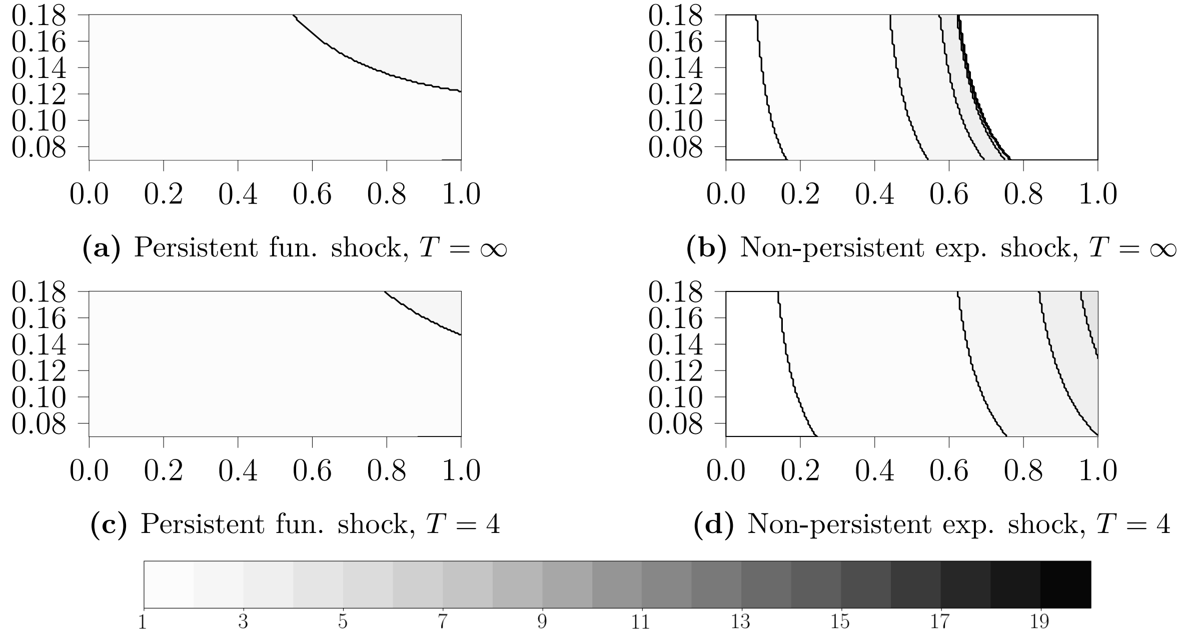
This section focuses on whether fiscal stimulus in the form of a temporary increase in government spending or cut in labor or consumption taxes can mitigate liquidity traps. In particular, assume that the government reacts to the start of the liquidity trap by implementing a stimulus package one period later. Forward-looking agents are further assumed to anticipate the coming stimulus package already in the first period of the liquidity trap. Finally, the stimulus package will be persistent, with auto-correlation coefficient 0.7.

In Section 4.1, I show that increases in government spending and consumption tax cuts reduce the duration of liquidity traps and can prevent deflationary spirals. Furthermore, I show that labor tax cuts, if anything, make liquidity traps worse. The mechanisms behind the latter result are illustrated in Section 4.2, while the workings of spending increases and consumption tax cuts are illustrated in Section 4.3. Finally Section 4.4 provides further intuition by discussing fiscal multipliers.

4.1 Durations under fiscal stimulus

To make the effectives of fiscal stimulus in mitigating a certain liquidity trap comparable across different shock sizes, the size of the stimulus package should vary with the size of the shock. As a benchmark, I therefore assume the size of the initial increase in government spending to be equal to the size of the initial shock hitting the economy. The sizes of the labor tax and consumption tax cuts will always be scaled compared to the spending increase size by respectively $\frac{1}{\bar{w}}$ and $\frac{1}{1-\bar{g}}$. This is to ensure that all stimulus measures have the same direct impact on the government's budget deficit, and hence are comparable.

In order to see the effects of fiscal stimulus on the durations of fundamentals-driven, mixed and expectations driven liquidity traps for different planning horizons, I reproduce Figures 1 and 2 under different stimulus packages. This is done for government spending



(a) Persistent fun. shock, $T = \infty$

(b) Non-persistent exp. shock, $T = \infty$

(c) Persistent fun. shock, $T = 4$

(d) Non-persistent exp. shock, $T = 4$

Figure 4: Durations of liquidity traps for different fractions of backward-looking agents (x-axis) and different shock sizes sizes (y-axis) in case of government spending increases. Panels (a) and (c) captures both fundamentals-driven and mixed liquidity traps for different planning horizons, while Panels (b) and (d) capture expectations-driven liquidity traps. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas in the top indicate deflationary spirals.

increases, consumption tax cuts and labor tax cuts in respectively Figures 4, 5 and 6. In all three figures, the top panels, (a) and (b), correspond to the infinite planning horizon case of Figure 1, while bottom panels (c) and (d) correspond to the case with $T = 4$ of Figure 2.

Starting with the fundamentals-driven liquidity trap in the most left part of panel (a), it can be seen in Figure 4 that, with government spending increases, longer lasting liquidity traps no longer arise. Instead, the liquidity trap always lasts for only one period, which indicates that the trap is resolved immediately in the period that the stimulus package is implemented (which is one period after the start of the liquidity trap). Looking at panel (a) of Figure 5, the same holds when fiscal stimulus takes the form of consumption tax cuts. Moreover, this result also holds for finite planing horizons, as can be seen in the most left part of panel (c) of the two figures. Under labor tax cuts, on the other hand, the durations of fundamentals driven liquidity traps become longer rather than shorter for both infinite and finite planning horizons (most left parts of panels (a) and (c) of Figure 6).

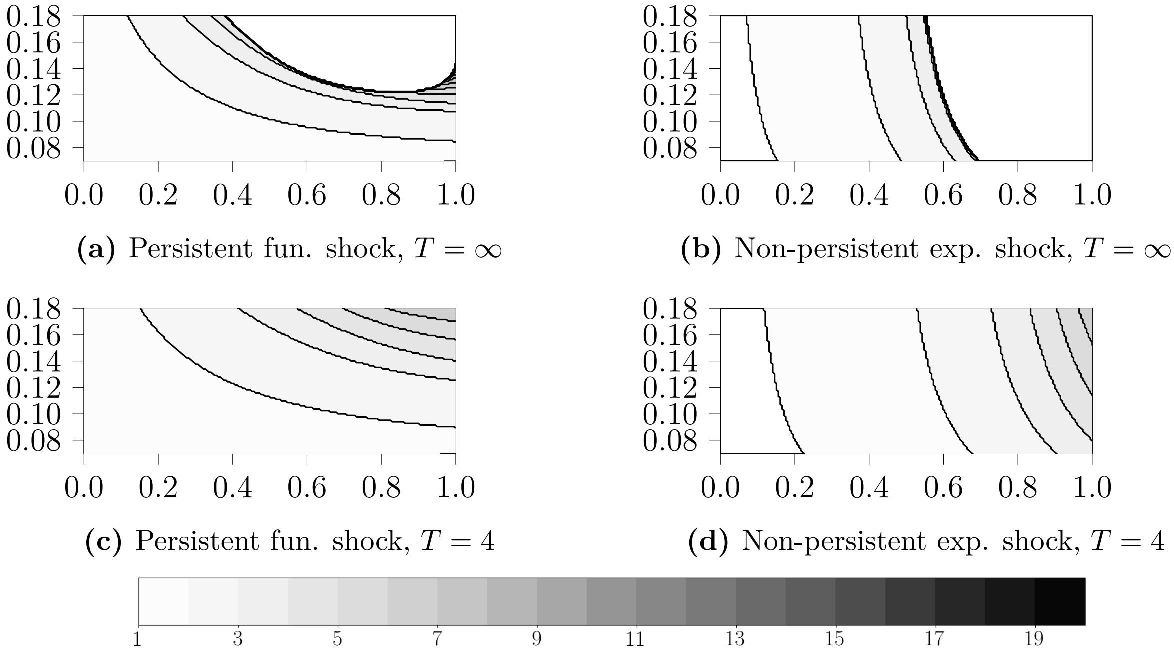
For mixed liquidity traps (the interior of panels (a) and (c) of the three figures) similar results are obtained: spending increases and consumption tax-cuts lead to shorter liquidity traps, while labor tax cuts do not. Note however, that spending increases seem to be somewhat more effective than consumption tax cuts. This is discussed in more detail in Section 4.3.

The next thing to note is that, for infinite planing horizons, deflationary spirals can still arise for very large shocks and relatively large fractions of backward-looking agents. These could however be eliminated with larger government spending increases or larger consumption tax cuts.[4]

Finally, note that, for finite planing horizons, government spending increases and consumption tax cuts seem to be especially effective for relatively large but not extremely large fractions of backward looking agents, e.g. around $\frac{3}{4}$. Here, deflationary spirals arose even for relatively small shock values in panel (a) of Figure 2. In panel (c) of Figures 4 and 5, on the other hand, deflationary spirals do not arise. Moreover, liquidity traps do not even seem to last particularly long for this range of fractions of backward-looking agents. The intuition for this result will be discussed in Sections 4.3 and 4.4.

Now, turn to the expectations-driven liquidity traps in panels (b) and (d) of Figures 4 and 5. Also here, spending increases and consumption tax cuts reduce the durations of liquidity traps compared to panel (b) of Figures 1 and 2. For finite planning horizons, the stimulus is, furthermore, again especially effective for a range of fractions of backward-looking agents around $\frac{3}{4}$, and for infinite planning horizons, deflationary spirals still arise.

[4] Results on the required stimulus size to fully eliminate liquidity traps are available on request.



(a) Persistent fun. shock, $T = \infty$

(b) Non-persistent exp. shock, $T = \infty$

(c) Persistent fun. shock, $T = 4$

(d) Non-persistent exp. shock, $T = 4$

Figure 5: Durations of liquidity traps for different fractions of backward-looking agents (x-axis) and different shock sizes sizes (y-axis) in case of consumption tax cuts. Panels (a) and (c) captures both fundamentals-driven and mixed liquidity traps for different planning horizons, while Panels (b) and (d) capture expectations-driven liquidity traps. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas in the top indicate deflationary spirals.

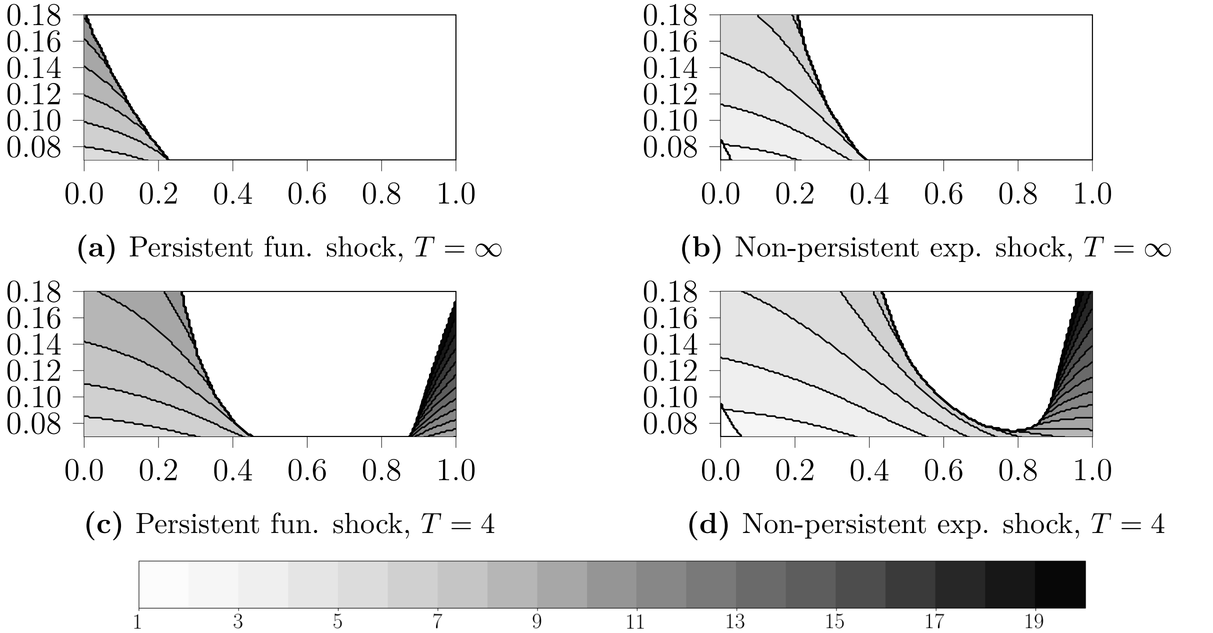


Figure 6: Durations of liquidity traps for different fractions of backward-looking agents (x-axis) and different shock sizes sizes (y-axis) in case of labor tax cuts. Panels (a) and (c) captures both fundamentals-driven and mixed liquidity traps for different planning horizons, while Panels (b) and (d) capture expectations-driven liquidity traps. Darker color shades indicate a longer duration of the liquidity trap. The fully white areas in the top indicate deflationary spirals.

These deflationary spirals could again be eliminated by larger stimulus packages, but the following is noteworthy. Under infinite planning horizons, fiscal stimulus can either almost completely eliminate a liquidity trap or the stimulus is not sufficient to prevent a deflationary spiral. Intermediate cases, where fiscal stimulus is able to prevent a deflationary spiral but results in a liquidity trap with a relatively long duration do not arise. That is, where finite planning horizons can lead to longer lasting liquidity traps from which the economy eventually recovers, this does not happen under infinite planning horizons, even when there is fiscal stimulus.

Finally, turning to panels (b) and (d) of Figures 6, it can be concluded that, also in expectations-driven liquidity traps, labor tax cuts are not an effective tool. The next section provides more insight in why that is the case.

4.2 The ineffectiveness of labor tax cuts

Above, it was found that labor tax cuts are not an effective tool for shortening any type of liquidity trap and cannot prevent deflationary spirals. This is a known result for fundamentals-driven liquidity traps under the infinite horizon rational expectations benchmark, since labor tax cuts are deflationary here.

Below, I show that this is also the case under heterogeneous expectations with finite planning horizons and identify an extra channel that makes anticipated labor tax cuts even less effective in mixed and expectations-driven liquidity traps.

Figure 7 depicts dynamics in a mixed liquidity trap with 25% backward-looking agents and a stimulus package in the form of labor tax cuts, which starts one period after the initial fundamental shock. As in Figure 3, solid curves depict medians, and dotted curves 0.05 and 0.95 quantiles.

First considering the blue curves without stimulus, it can be observed that output falls persistently due to the persistent negative preference shock. The fall in aggregate demand leads to a fall in labor demand, causing a drop in wages (bottom-right panel). This implies lower marginal costs for firms and a persistent drop in inflation. Moreover, all variables fall even more due to backward-looking agents that become pessimistic after observing low output and inflation, and due to forward-looking agents anticipating this. The latter mechanism is similar to the one in the expectations-driven liquidity trap described in Section 3.4.

The red curves in Figure 7 depict the case where labor taxes are cut in period 2. This leads to an increase in labor supply (due to the substitution effect). As a consequence, wages fall more to equate labor supply and labor demand. Lower wages, in turn, lead to lower marginal costs for firms and lower inflation.

Since the interest rate is stuck at the zero lower bound, lower inflation implies higher real interest rates. Lower expected future inflation hence puts downward pressure on the

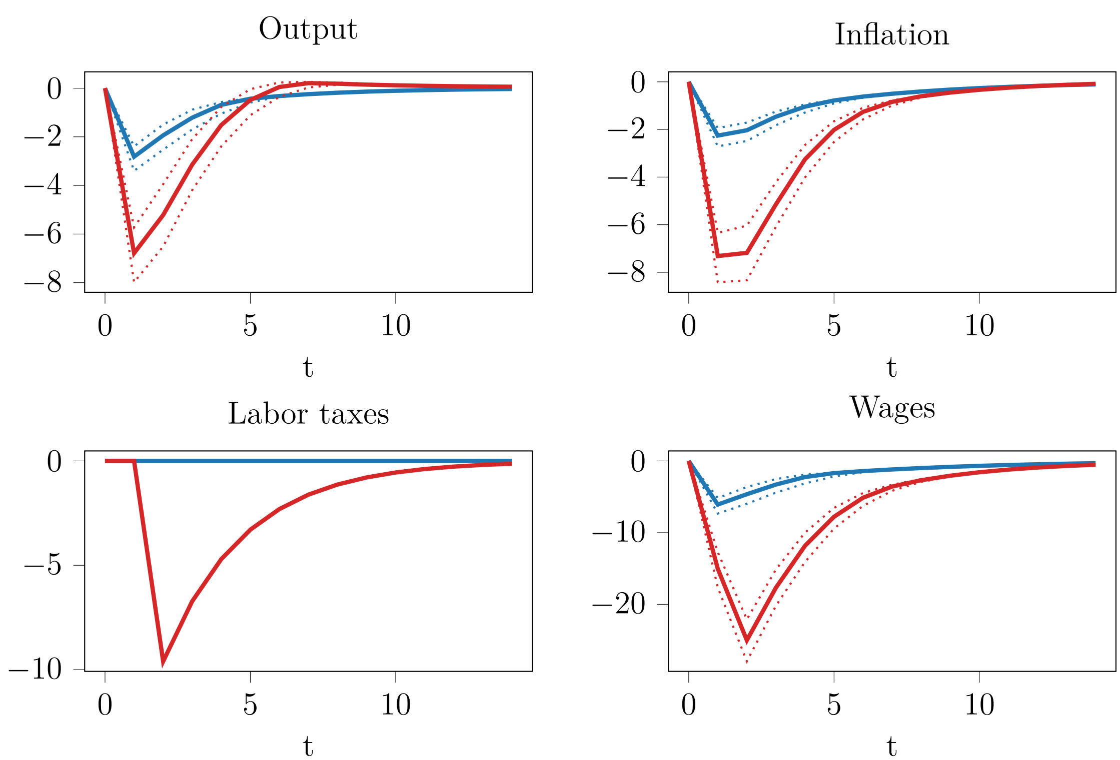


Figure 7: Mixed liquidity trap for $T = 4$, $\hat{\xi}_1 = -0.08$ and 25% backward-looking agents. Red curves depict the case of fiscal stimulus in the form of labor tax cuts, while blue curves correpsond to the case of no stimulus. Solid curves are median impulse responses and dotted curves depict 0.05 and 0.95 quantiles.

consumption of forward-looking households. Consumption of backward-looking households will be negatively effected slightly later, when they have observed the drop in inflation. Hence, labor tax cuts are not only deflationary, but also are not able to significantly increase output. In the end, the liquidity trap is made worse rather than better by the stimulus package and its duration is increased.

It follows from the above that the effects of labor tax cuts on inflation and output depend on the extend to which labor supply and hence wages are affected. The precise dynamics that arise after a labor tax cut hence depend on the calibration of the parameters in the utility function, η and σ. For example, if the Frisch elasticity of labor supply, η, were lower, labor supply would react less to a change in labor taxes, resulting in a smaller drop in wages and inflation. However, the result that labor tax cuts are deflationary and make liquidity traps last long rather than shorter holds for all reasonable calibrations of σ and η.

This main mechanism renders labor taxes ineffective for any fraction of backward-looking agents and any horizon. It holds under mixed and expectations-driven liquidity traps as well as fundamentals-driven liquidity traps. In Figure 7 there is however an additional channel that makes anticipated labor tax cuts less effective in a mixed liquidity trap. This channel is not present in the homogeneous expectations benchmark.

When forward-looking agents anticipate lower future prices and consumption and higher future real interest rates, they cut prices and consumption already in the period *before* the start of the consolidation package. In a fundamentals-driven liquidity trap there is not much endogenous persistence, implying that the effect of this initial anticipation on inflation and output in later periods is small. In contrast, in a mixed liquidity trap (or expectations-driven liquidity trap) output and inflation in the period where the stimulus package is actually implemented become considerably lower due to these anticipation effects. This is because backward-looking agents observe the lower prices and output caused by the anticipation of forward-looking agents in the previous period and adjust their own expectations accordingly. As a consequence, they reduce consumption and prices more.

In a mixed or expectations-driven liquidity trap with both forward-looking and backward-looking agents, anticipated consolidations hence are even less effective in the period of implementation than in a fundamentals-driven liquidity trap with homogeneous rational expectations.

4.3 Effective fiscal stimulus under heterogeneous-expectations

Next, I consider in detail the other two fiscal instruments: government spending increases and consumption tax cuts. In Section 4.1, we saw that fiscal stimulus with one of these measures can considerably reduce the length of a liquidity trap and even prevent a deflationary spiral. Below, I show why this is the case. I do so for a mixed liquidity trap where half of the agents are forward-looking and half of the agents are backward-looking. Intuitions for different fractions of backward-looking agents and for expectations-driven liquidity traps are similar to those discussed below.

In Figure 8, the case of no fiscal stimulus is plotted in blue. Dynamics here are similar to those in Figure 7, but the liquidity trap is somewhat worse due to the higher fraction of backward-looking agents. Especially visible is the larger drop in inflation in period 2, which arises due to backward-looking agents having lower inflation expectations because of the observed drop in period 1 inflation. The liquidity trap also lasts longer then the blue case in Figure 7. These worse conditions allow the effectiveness of spending increases and consumption tax cuts to be illustrated even more convincingly.

The green curves in Figure 8 plot the case of an anticipated spending increase package that starts one period after the start of the liquidity trap (i.e., in period 2). Higher government spending leads to higher labor demand and higher wages. This leads to higher marginal costs and inflation. This, in turn, leads to lower real interest rates since the

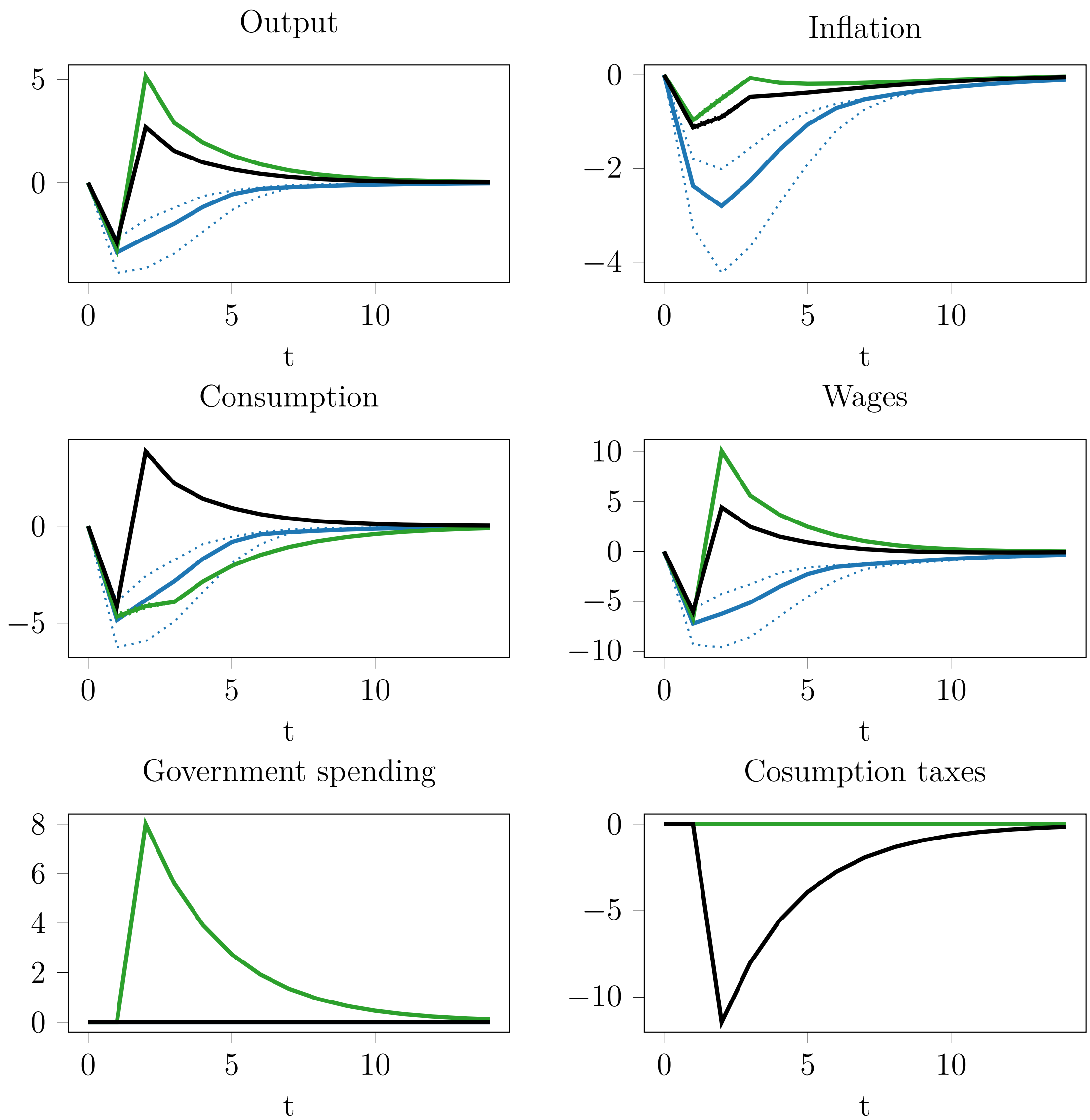


Figure 8: Mixed liquidity trap for $T = 4$, $\hat{\xi}_1 = -0.08$ and 50% backward-looking agents. Green and black curves depict the cases of fiscal stimulus in the form of respectivly governemtn spending increases and consumption tax cuts, while blue curves correpsond to the case of no stimulus. Solid curves are median impulse responses and dotted curves depict 0.05 and 0.95 quantiles.

nominal interest rate is stuck at the zero lower bound. Therefore, there is very little crowding out and consumption does not fall by much compared to the blue curves (bottom-left panel). Hence the higher government spending leads to a considerable increase in output.

Finally, the black curves depict the case of consumption tax cuts that start in period 2. Here, households consume more because of the lower taxes. Hence, labor demand goes up, and so do wages. This leads to inflation and lower real interest rates, further increasing current consumption and output.

The above mechanisms are also present in fundamentals-driven liquidity trap. In a fundamentals-driven liquidity trap, a stimulus package that is large an persistent enough can so off-set the negative fundamental shock and immediately eliminate the liquidity trap. For a given shock size, however, mixed liquidity traps feature lower output and inflation and a longer liquidity trap. So why is it that a fiscal stimulus package of the same size is still so effective in ending the liquidity trap?

If increases in government spending or cuts in consumption taxes increase output and inflation, then this counteracts the fundamental shock, as in the homogeneous case. But, additionally, higher inflation and output will cause expectations of backward-looking agents to be considerably higher one period later. That is, the stimulus, not only offsets the fundamentals part of the mixed liquidity trap, but after one period also the expectations part. Since forward-looking agents anticipate this ending of both the fundamentals and expectations part of the liquidity trap, they will have much higher output and inflation expectations already in the two periods before, which makes the fiscal stimulus even more effective.

Similar arguments apply to expectations-driven liquidity traps with both backward-looking and forward-looking agents, explaining why fiscal stimulus is an highly effective tool in that case as well. This is discussed in more detail in the next section.

As in the previous section, the exact effects of a fiscal stimulus depend on the utility function of the household. Particularly interesting for the comparison of spending increases versus consumption tax cuts is the inverse of the intertemporal elasticity of substitution: σ. When this parameter is calibrated at a lower value, households respond more with their current period consumption to any change in expectations or in income. This means that consumption falls more when government spending is increased and that consumption increases more when consumption taxes are lowed. Hence, with a lower value of σ, government spending increases are relatively less effective in mitigating a liquidity trap, while consumption tax cuts are relatively more effective. In particular, for a value of $\sigma = 1$, consumption taxes are almost as effective as government spending increases in shortening liquidity traps and preventing deflationary spirals.

4.4 Fiscal multipliers

To gain more insight in how heterogeneous expectations influence the effectiveness of fiscal stimulus in a liquidity trap, I now turn to fiscal multipliers. Panel (a) and (b) of Table 1 presents multipliers for different fractions of backward-looking agents in case of a persistent fundamental shock, while panel (c) considers expectations-driven liquidity traps. I present both impact multipliers, and cumulative multipliers after 12 periods (last 3 columns of both panels). The multipliers are calculated by considering a very small change in the fiscal instrument (an increase of 0.0001). This way, the stimulus never changes the duration of a liquidity trap, which would have made multipliers less comparable. Multipliers are then calculated following Mountford and Uhlig (2009) and Bi et al. (2013) as

$$\Gamma_{t+k} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(Y_{t+j}^{s} - Y_{t+j}^{ns} \right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1} \right) \left(x_{t+j}^{s} - x_{t+j}^{ns} \right), \tag{27}$$

where r_t is the gross interest rate, and x_t denotes the type of fiscal stimulus. $x_t = G_t$ in case of government spending increases, $x_t = \tau_t^c \bar{C}$ in case of consumption tax cuts, and $x_t = \tau_t^l \bar{w} \bar{H}$ in case of labor tax cuts.[5] Y_t^s and x_t^s indicate values taken when there is fiscal stimulus and Y_t^{ns} and x_t^{ns} indicate values that would have occurred in the absence of fiscal stimulus.

The first row of panel (b) corresponds to the case of a fundamentals-driven liquidity trap when $T = 4$, with a shock size of 0.08. For comparison, the infinite horizon case is given in the first row of panel (a) of Table 1. In both cases, impact multipliers of government spending are close to 1, indicating that government spending increases can successfully increase output. The consumption tax multipliers are somewhat smaller (in absolute value), but also consumption tax cuts will considerably increase output. The impact multipliers for labor taxes, on the other hand, are approximately zero, confirming that labor tax cuts cannot increase output in the short-run. Cumulative labor tax multipliers after 12 periods are negative, but still relatively small, indicating a very minor positive medium-run effect of labor tax cuts. Medium-run cumulative multipliers of government spending and consumption taxes are similar to the respective impact multipliers, but somewhat smaller.

Before turning to the mixed liquidity traps, lets first consider the multipliers in expectation-driven liquidity traps in panel (c) of Table 1. For small fractions of backward-looking agents, no expectations-driven liquidity traps of more than 1 period arise. For large fractions of backward-looking agents and an infinite planning horizon, deflationary spirals arise for all shock sizes so that no meaningful multipliers can be obtained there. Therefore, panel

[5] I multiply labor taxes by $\bar{w}\bar{H}$ and consumption taxes by $\bar{C}$ to get a change in tax income due to a changed tax rate, rather than the change in the tax rate itself. This facilitates compatibility with changes in government spending. Using the definitions of (log)-linearized variables, I then calculate multipliers as follows. $\Gamma_{t+k} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1}\right)\left(\hat{Y}_{t+j}^{s} - \hat{Y}_{t+j}^{ns}\right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1}\right)\left(\tilde{g}_{t+j}^{s} - \tilde{g}_{t+j}^{ns}\right)$ for government spending, $\Gamma_{t+k} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1}\right)\left(\hat{Y}_{t+j}^{s} - \hat{Y}_{t+j}^{ns}\right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1}\right)\left((1-\bar{g})(\tilde{\tau}_{t+j}^{c,s} - \tilde{\tau}_{t+j}^{c,ns})\right)$ for consumption taxes, and $\Gamma_{t+k} = \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1}\right)\left(\hat{Y}_{t+j}^{s} - \hat{Y}_{t+j}^{ns}\right) / \sum_{j=0}^{k} \left(\prod_{i=0}^{j} r_{t+i}^{-1}\right)\left(\bar{w}(\tilde{\tau}_{t+j}^{l,s} - \tilde{\tau}_{t+j}^{l,ns})\right)$ for labor taxes. In computing the multipliers, I use the realized real interest rates under fiscal stimulus along the transition path.

Panel (a): $T = \infty$ and persistent negative fundamental shock (of size 0.08)

	Impact multiplier			Medium-run multiplier (12 per.)		
Frac. BL	Gov. spen.	Cons. tax	Labor tax	Gov. spen.	Cons. tax	Labor tax
0	0.92	-0.56	0.02	0.74	-0.46	-0.14
0.25	1.50	-0.92	0.51	1.12	-0.62	0.19

Panel (b): $T = 4$ and persistent negative fundamental shock (of size 0.08)

	Impact multiplier			Medium-run multiplier (12 per.)		
Frac. BL	Gov. spen.	Cons. tax	Labor tax	Gov. spen.	Cons. tax	Labor tax
0	0.98	-0.61	0.01	0.79	-0.49	-0.13
0.25	1.31	-0.82	0.23	1.01	-0.63	0.01
0.5	2.10	-1.32	0.71	1.76	-1.11	0.47
0.75	2.43	-1.53	0.88	2.95	-1.87	1.17
0.875	1.34	-0.83	0.20	1.74	-1.10	0.40
1	1.00	-0.62	-0.01	1.28	-0.81	0.11

Panel (c): $T = 4$ and non-persistent negative shock to expectations (of size 0.11)

	Impact multiplier			Medium-run multiplier (12 per.)		
Frac. BL	Gov. spen.	Cons. tax	Labor tax	Gov. spen.	Cons. tax	Labor tax
0.6	1.14	-0.71	0.11	0.92	-0.57	-0.07
0.7	1.34	-0.83	0.22	1.19	-0.75	0.09
0.8	1.43	-0.89	0.27	1.54	-0.97	0.30
0.9	1.28	-0.80	0.16	1.98	-1.25	0.54
1	1.00	-0.62	-0.01	1.56	-0.98	0.27

Table 1: Impact multipliers and 12-period cumulative multipliers for different fiscal instruments and different types of liquidity traps.

(c) depicts multipliers only for $T = 4$ and only for larger fractions of backward-looking agents. The shock size is set to 0.11.

Looking at the impact spending and consumption tax multipliers in panel (c), it can be seen that these are larger than those in the fundamentals-driven liquidity trap. Moreover, multipliers initially become larger as the fraction of backward-looking agents increases, but then become smaller again when the fraction of backward-looking agents becomes very large. The same holds for the cumulative multipliers in the right part of the panel, but here the largest multipliers are reached at fractions of around 0.9 rather than 0.8.

The intuition for these results comes from the interaction of the expectations of the two types of agents. Since increases in government spending and consumption tax cuts increase inflation and output, these measures raise expectations of backward-looking agents one period later and thus counteract the driving force of the liquidity trap. The larger the fraction of backward-looking agents, the more output will be raised by this in later periods and the larger the medium-run cumulative multiplier. Forward-looking agents anticipate this and turn more optimistic about the future already at the start of the liquidity trap. They, therefore, will have higher prices and consumption already at the period where the stimulus package is first implemented, which increases the impact multiplier. This leads to a feedback mechanism, where the higher output and inflation on impact cause expectations of backward-looking agents to be higher one period later, which again implies a larger cumulative multiplier and through the expectations of forward-looking agents an even larger impact multiplier.

The size of the impact multiplier hence depends on the size of the medium-run cumulative multiplier, but also on the fraction of forward-looking agents. When the fraction of backward-looking agents is 0.9, there are too little forward-looking agents in the economy to let the high cumulative multiplier be translated in to a very high impact multiplier. Finally, when the there are only backward-looking agents, the above feedback mechanism completely disappears. The impact multipliers of government spending and consumption taxes are now comparable in size to the case of a fundamentals-driven liquidity trap, and the cumulative multipliers are also somewhat lower again.

Now, turn to the mixed liquidity trap cases in rows 2 - 5 of panel (b). As in panel (c), the impact multipliers for government spending and consumption taxes first become considerably larger and then smaller again as the fraction of backward-looking agent grows. However, the cumulative multiplier now co-moves more with the impact multiplier and also has its peek around 0.75. Moreover, both impact and cumulative multipliers are considerably larger here than under expectations-driven liquidity traps.

The intuition for these findings is the feedback mechanism described in 4.3. That is, the interaction of forward-looking agents, who expect both the fundamentals part and the expectations part of the liquidity trap to be reduced by the stimulus, with backward-looking agents, who respond positively in later periods to price and consumption increases of forward-looking agents. This can lead to very large positive effects of fiscal stimulus.

The reason that the largest cumulative and impact multiplier are reached at a smaller fraction of backward-looking agents than in the expectations-driven liquidity trap is that in a mixed liquidity trap output and inflation become considerably lower when there also is a large fraction of forward-looking agents (we saw this in the blue curves in Figure 8). Hence, there is more potential for the feedback mechanism to raise output and inflation. This is not so much the case in the expectations driven liquidity trap where the fundamentals part of the trap is absent.

In the second row of panel (a) it can be seen that mixed liquidity lead to larger government spending and consumption tax multipliers than fundamentals-driven liquidity traps also for the case of infinite planning horizons. Meaningful multipliers for larger fractions of backward-looking agents cannot be obtained here since deflationary spirals then arise without stimulus.

Finally, turn to the labor tax multipliers in expectations-driven and mixed liquidity traps. These are (almost) always positive, indicating that labor tax cuts cannot increase output on impact or in the medium run. Moreover, the size of the multiplier as the fraction of backward-looking agents is varied follows a similar pattern as that of the government spending multiplier. This is because the feedback mechanism that negatively effects anticipated labor tax cuts (discussed in Section 4.2) is also largest when there is both a large fraction of backward-looking agents and a significant fraction of forward-looking agents.

5 Discussion of implicit long run expectations in value functions

In this paper, I have mainly focused on liquidity traps in case of finite planning horizons. Agents with finite planning horizons only have the cognitive ability to make a detailed consumption and pricing plan based on expectations for T periods into the future. However, they also care about what will happen after that by using a value function to asses the value of having certain amount of wealth or a certain relative price in terms of utility in the periods outside their planning horizon. This value function can be motivated as being based on experience over a long time span, and is generally not supposed to change due to short run fluctuations in the economy.

However, one could argue that liquidity traps are exceptional times and that, in a severe liquidity trap, pessimism not only affects expectations about short run fluctuations, but also causes the valuation of states in the long run to suddenly change. This could, e.g., happen if agents no longer believe that the economy will eventually return to its normal state, but that instead variables as output and inflation will be permanently lower. This kind of long-run pessimism would have additional negative effects on output and inflation, leading to worse liquidity traps.

To see this, first consider the firm problem. If the value function of firms would be allowed to adjust in a time of long-run pessimism, this would be reflected in values for λ, Y, mc and Π in (2.2) that deviate from their steady state values. In particular, we could make these long run believes (denoted with superscript LR) time varying variables and write

$$\tilde{V}(r) = \frac{1}{1-\omega\beta(\Pi^{LR})^{\theta-1}}\lambda_t^{LR}\left(\frac{r}{\Pi_t^{LR}}\right)^{1-\theta}Y_t^{LR} - \frac{1}{1-\omega\beta(\Pi_t^{LR})^{\theta}}\lambda_t^{LR}\left(\frac{r}{\Pi_t^{LR}}\right)^{-\theta}Y_t^{LR}mc_t^{LR}. \tag{28}$$

In Appendix D it is shown that, in this case, extra terms arise in the Phillips curve, affecting inflation as follows. Long-run believes about inflation affect current inflation with coefficient $\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\left(\frac{\theta(c_1)^{T+1}}{1-c_1} - \frac{(\theta-1)(c_2)^{T+1}}{1-c_2}\right) > 0$. Further, $\hat{Y}_t^{LR}$ and $\hat{\lambda}_t^{LR}$ both have a coefficient $\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\left((c_1)^{T+1} - (c_2)^{T+1}\right) > 0$, and $\hat{mc}_t^{LR}$ has a coefficient $\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(c_1)^{T+1} > 0$. Here, $c_1 = \omega\beta\bar{\Pi}^{\theta}$ and $c_2 = \omega\beta\bar{\Pi}^{\theta-1}$.

In the households' value function, long-run pessimism would reflect in believes about long run income, Λ, and long run inflation. In Appendix D it is shown that when these two parameters in the value function are made time varying, two additional terms in the output equations show up. In particular, long-run believes about income affect current output with coefficient $\frac{\beta^{T+1}}{1-\beta}\frac{u_b\bar{\Lambda}}{\bar{C}(1+\bar{\tau}^c)\rho} > 0$ and long-run believes about inflation affect current output with coefficient $-\frac{\beta^{T+1}}{\rho}(\bar{b} - \frac{u_b}{(1-\beta)\sigma})$. Composite parameters are defined in Appendix B.

From the above it is clear that pessimism about the long-run state of the economy, through the value function, can depress output and inflation, making liquidity traps worse. This could then also create extra scope for effective fiscal stimulus, since long run pessimism may (partly) disappear if agents expect or experience a fiscal stimulus package. A full analysis of this issue is, however, beyond the scope of the paper.

6 Conclusion

I present a New Keynesian model with two forms of bounded rationality. First of all, while one fraction of agents is forward-looking and has rational expectations, another fraction of agents forms expectations in a backward-looking manner, based on the most recently observed state of the economy. They expect a similar economic situation to continue in the short run, but expect mean reversion to the target steady state in the medium to long run. Secondly, all agents in the economy have a finite planning horizon and are not able to base their consumption and pricing decisions upon considerations and expectations about the infinite future.

The presence of backward-looking agents in the economy can result in liquidity traps of multiple periods that are fully (expectations-driven liquidity trap) or partly (mixed liquidity trap) driven by expectations. The former type arises after a single negative shock to inflation and output expectations of backward-looking agents, while the latter arises because of a persistent fundamental shock. The duration of these liquidity trap crucially depends on agents' planning horizons. For infinite horizons, the liquidity trap either lasts at most three or four periods, or the economy ends up in a deflationary spiral. When planning horizons are short, on the other hand, deflationary spirals become less likely, and instead long lasting liquidity traps from which the economy eventually recovers can arise.

I show that fiscal stimulus in the form of spending increases or cuts in consumption taxes is very effective in reducing the duration of liquidity traps and preventing deflationary spirals. Labor tax cuts on the other hand, are deflationary and are not an effective tool in any of the liquidity traps studied in this paper. Spending increases and consumption tax cuts increase labor demand and wages, resulting in inflationary pressures. Fiscal multipliers of these instruments are largest when the majority of agents is backward-looking but there also is a significant fraction of forward-looking agents in the economy. In that case, a feedback mechanism arises, where forward-looking expect the stimulus to lead to higher future output and inflation and increase current consumption and prices. This is later observed by backward-looking agents, causing them to also increase consumption and prices, which is again anticipated by forward-looking agents. This feedback is strongest in mixed liquidity traps, but also leads to large multipliers in expectations-driven liquidity traps.

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A Steady state

In this section, the steady state of the non-linear model is derived, where the preference shock is assumed to be constant at $\xi = 1$.

From the consumer Euler equation it follows that in this steady state we must have

$$\frac{1+\bar{i}}{\bar{\Pi}} = \frac{1}{\beta} \tag{29}$$

Furthermore, from (58) it follows that

$$\bar{H} = \bar{s}\bar{Y} \tag{30}$$

Next, we can solve the steady state aggregate resource constraint, (21), for consumption, and write

$$\bar{C} = \bar{Y}(1-\bar{g}) \tag{31}$$

Plugging in these steady state labor and consumption levels in the steady state version of the optimal labor/consumption trade off gives

$$\bar{mc} = \bar{w} = \frac{\bar{s}^{\eta}\bar{Y}^{\eta+\sigma}(1-\bar{g})^{\sigma}(1+\bar{\tau}^{c})}{1-\bar{\tau}^{l}} \tag{32}$$

So that steady state output can be written as

$$\bar{Y} = \left(\frac{\bar{mc}(1-\bar{\tau}^{l})}{\bar{s}^{\eta}(1-\bar{g})^{\sigma}(1+\bar{\tau}^{c})}\right)^{\frac{1}{\eta+\sigma}} \tag{33}$$

For the relative optimal price, we write (53) as

$$\bar{d} = \left(\frac{1-\omega\Pi^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}} \tag{34}$$

For price dispersion, we then write

$$\bar{s} = \frac{(1-\omega)}{1-\omega\bar{\Pi}^{\theta}}\bar{d}^{-\theta} = \frac{(1-\omega)}{1-\omega\bar{\Pi}^{\theta}}\left(\frac{1-\omega\Pi^{\theta-1}}{1-\omega}\right)^{\frac{\theta}{\theta-1}} \tag{35}$$

Evaluating (47) at the steady state gives

$$\bar{mc} = \bar{d}\frac{\theta-1}{\theta}\frac{\sum_{s=0}^{T}\omega^{s}\beta^{s}\bar{\Pi}^{s(\theta-1)} + \frac{(\omega\beta\bar{\Pi}^{\theta-1})^{T+1}}{1-\omega\beta\bar{\Pi}^{\theta-1}}}{\sum_{s=0}^{T}\omega^{s}\beta^{s}\bar{\Pi}^{s(\theta)} + \frac{(\omega\beta\bar{\Pi}^{\theta})^{T+1}}{1-\omega\beta\bar{\Pi}^{\theta}}} = \left(\frac{1-\omega\Pi^{\theta-1}}{1-\omega}\right)^{\frac{1}{1-\theta}}\frac{\theta-1}{\theta}\frac{(1-\omega\beta\Pi^{\theta})}{(1-\omega\beta\Pi^{\theta-1})} \tag{36}$$

Firm profits we can write as

$$\bar{\Xi} = (1-\bar{w}\bar{s})\bar{Y} \tag{37}$$

Then we turn to the government budget constraint. In steady state (20) reduces to

$$\frac{\beta\bar{b}}{\bar{\Pi}} = (1+\bar{\tau}^{c})\bar{g} - \bar{\tau}^{l}\bar{w}\bar{s} - \bar{\tau}^{c} - \frac{\bar{LS}}{\bar{Y}} + \frac{\bar{b}}{\bar{\Pi}}, \tag{38}$$

which gives

$$\bar{b} = \bar{\Pi}\frac{(\bar{\tau}^{l}\bar{w}\bar{s} + \bar{\tau}^{c} + \frac{\bar{LS}}{\bar{Y}} - (1+\bar{\tau}^{c})\bar{g})}{1-\beta}, \tag{39}$$

B Log-linearized model

In this Section, I log-linearize the model equations around the steady state.

B.1 Households

The log linearized optimality conditions of the households (including budget constraints) are given by

$$\hat{C}_{\tau}^{i} = \hat{C}_{\tau+1}^{i} - \frac{1}{\sigma}(i_{\tau} - \hat{\pi}_{\tau+1} + \xi_{\tau+1} - \xi_{\tau} - \frac{\tilde{\tau}_{\tau+1}^{c} - \tilde{\tau}_{\tau}^{c}}{1+\bar{\tau}^{c}}), \qquad \tau = t, t+1, ...t+T-1 \tag{40}$$

$$\tilde{b}^i_{t+T+1} = \frac{u_b}{1-\beta}\hat{C}^i_{t+T} + \frac{u_b}{(1-\beta)\sigma}\frac{\tilde{\tau}^c_{t+T}}{1+\bar{\tau}^c} + \frac{u_b}{(1-\beta)\sigma}E^i_t i_{t+T} - \frac{u_b}{(1-\beta)\sigma}\xi_{t+T}, \tag{41}$$

$$\eta\hat{H}^i_\tau = -\sigma\hat{C}^i_\tau - \frac{\tilde{\tau}^c_\tau}{1+\bar{\tau^c}} - \frac{\tilde{\tau}^l_\tau}{1-\bar{\tau}^l} + \hat{w}_\tau, \qquad \tau = t, t+1, ...t+T \tag{42}$$

$$\begin{aligned}\tilde{b}^i_{\tau+1} =& \frac{\bar{w}\bar{s}\bar{\Pi}}{\beta}((1-\bar{\tau}^l)(E^i_t\hat{w}_\tau + \hat{H}^i_\tau) - E^i_t\tilde{\tau}^l_\tau) + \frac{1}{\beta}\tilde{b}^i_\tau + \bar{b}(\hat{i}_\tau - \frac{1}{\beta}E^i_t\hat{\pi}_\tau) + \frac{\bar{\Xi}\bar{\Pi}}{\bar{Y}\beta}E^i_t\hat{\Xi}_\tau \\ &- \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\beta}E^i_t\hat{LS}_\tau - \frac{(1-\bar{g})\bar{\Pi}}{\beta}\left((1+\bar{\tau}^c)\hat{C}^i_\tau + \tilde{\tau}^c_\tau\right), \qquad \tau = t, t+1, ...t+T\end{aligned} \tag{43}$$

with

$$u_b = (1-\bar{g})(1+\bar{\tau}^c)\bar{\Pi} \tag{44}$$

where it is used that $\bar{H} = \bar{s}\bar{Y}$, $\frac{\bar{C}}{\bar{Y}} = 1 - \bar{g}$ and $\frac{\Lambda}{1+\bar{\tau}^c} + \frac{1-\beta}{1+\bar{\tau}^c}\frac{\bar{Y}\bar{b}}{\Pi} = \bar{C}$

Iterating the budget constraint T periods, and using the first order conditions of the household, the following equation can be derived, that describes a households optimal consumption decision in period t.

$$
\begin{aligned}
&\left(\frac{\beta^{T+1}}{1-\beta}u_b + \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\frac{1-\beta^{T+1}}{1-\beta}\right)\hat{C}_t^i = \\
&\tilde{b}_t^i + \bar{w}\bar{s}\bar{\Pi}(1-\bar{\tau}^l)\sum_{s=0}^{T}\beta^s\left((1+\frac{1}{\eta})(E_t^i\hat{w}_{t+s} - \frac{E_t^i\tilde{\tau}_{t+s}^l}{1-\bar{\tau}^l}) - \frac{1}{\eta}\frac{E_t^i\tilde{\tau}_{t+s}^c}{1+\bar{\tau^c}}\right) + \frac{\bar{\Xi}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t^i\hat{\Xi}_{t+s}) \\
&-\frac{\bar{LS}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(E_t^i\hat{LS}_{t+s}) - \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\sum_{j=0}^{s-1}\frac{1}{\sigma}(E_t^i i_{t+j} - E_t^i\hat{\pi}_{t+j+1}) \\
&-\left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\left(\frac{1}{\sigma}(E_t^i\xi_{t+s} - E_t^i\xi_t - \frac{E_t^i\tilde{\tau}_{t+s}^c - E_t^i\tilde{\tau}_t^c}{1+\bar{\tau^c}})\right) \\
&-(1-\bar{g})\bar{\Pi}\sum_{s=0}^{T}\beta^s\left(E_t^i\tilde{\tau}_{t+s}^c\right) + \bar{b}\sum_{s=0}^{T}\beta^s(\beta E_t^i\hat{i}_{t+s} - E_t^i\hat{\pi}_{t+s}) \qquad (45) \\
&-\frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\sum_{j=0}^{T-1}(E_t^i i_{t+j} - E_t^i\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}E_t^i i_{t+T} + \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\xi_t - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\frac{\tilde{\tau}_t^c}{1+\bar{\tau}^c}
\end{aligned}
$$

Aggregating this equation over all households yields an expression for aggregate consumption as a function of aggregate expectations about aggregate variables, only.

$$
\begin{aligned}
&\left(\frac{\beta^{T+1}}{1-\beta}u_b + \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\frac{1-\beta^{T+1}}{1-\beta}\right)\hat{C}_t = \\
&\tilde{b}_t + \bar{w}\bar{s}\bar{\Pi}(1-\bar{\tau}^l)\sum_{s=0}^{T}\beta^s\left((1+\frac{1}{\eta})(\bar{E}_t\hat{w}_{t+s} - \frac{\bar{E}_t\tilde{\tau}_{t+s}^l}{1-\bar{\tau}^l}) - \frac{1}{\eta}\frac{\bar{E}_t\tilde{\tau}_{t+s}^c}{1+\bar{\tau^c}}\right) + \frac{\bar{\Xi}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{\Xi}_{t+s}) \\
&-\frac{\bar{LS}\bar{\Pi}}{\bar{Y}}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{LS}_{t+s}) - \left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\sum_{j=0}^{s-1}\frac{1}{\sigma}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) \\
&-\left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\sum_{s=1}^{T}\beta^s\left(\frac{1}{\sigma}(\bar{E}_t\xi_{t+s} - \bar{E}_t\xi_t - \frac{\bar{E}_t\tilde{\tau}_{t+s}^c - \bar{E}_t\tilde{\tau}_t^c}{1+\bar{\tau^c}})\right) \\
&-(1-\bar{g})\bar{\Pi}\sum_{s=0}^{T}\beta^s\left(\bar{E}_t\tilde{\tau}_{t+s}^c\right) + \bar{b}\sum_{s=0}^{T}\beta^s(\beta\bar{E}_t\hat{i}_{t+s} - \bar{E}_t\hat{\pi}_{t+s}) \qquad (46) \\
&-\frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\sum_{j=0}^{T-1}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\bar{E}_t i_{t+T} + \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\xi_t - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma}\frac{\tilde{\tau}_t^c}{1+\bar{\tau}^c}
\end{aligned}
$$

B.2 Firms

Equation (17) can be written as

$$\frac{p_t^*(j)}{P_t}\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\frac{P_{t+s}}{P_t}\right)^{\theta-1} Y_{t+s} + \frac{(\omega\beta)^{T+1}}{1-\omega\beta\bar{\Pi}^{\theta-1}}\bar{Y}\bar{\lambda}\left(\frac{\bar{\Pi}P_{t+T}}{P_t}\right)^{\theta-1}\right] \tag{47}$$

$$=\frac{\theta}{\theta-1}\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\frac{P_{t+s}}{P_t}\right)^{\theta} Y_{t+s} mc_{t+s} + \frac{(\omega\beta)^{T+1}}{1-\omega\beta\bar{\Pi}^{\theta}}\bar{Y}\bar{\lambda}\bar{mc}\left(\frac{\bar{\Pi}P_{t+T}}{P_t}\right)^{\theta}\right]$$

Eliminating prices, we can instead write the equation in terms of $d_t(j) = \frac{p_t^*(j)}{P_t}$ and in terms of inflation as

$$d_t(j)\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\prod_{j=1}^{s}\Pi_{t+j}\right)^{\theta-1} Y_{t+s} + \frac{(\omega\beta)^{T+1}\bar{\Pi}^{\theta-1}}{1-\omega\beta\bar{\Pi}^{\theta-1}}\bar{Y}\bar{\lambda}\left(\prod_{j=1}^{T}\Pi_{t+j}\right)^{\theta-1}\right]$$

$$=\frac{\theta}{\theta-1}\left[\tilde{E}_t^j \sum_{s=0}^{T} \omega^s \beta^s \frac{\xi_{t+s}}{1+\tau_{t+s}^c}\left(C_{t+s}^j\right)^{-\sigma}\left(\prod_{j=1}^{s}\Pi_{t+j}\right)^{\theta} Y_{t+s} mc_{t+s} + \frac{(\omega\beta)^{T+1}\bar{\Pi}^{\theta}}{1-\omega\beta\bar{\Pi}^{\theta}}\bar{Y}\bar{\lambda}\bar{mc}\left(\prod_{j=1}^{T}\Pi_{t+j}\right)^{\theta}\right], \tag{48}$$

Log linearizing (48) gives

$$\begin{aligned}\hat{d}_t(j) =& \tilde{E}_t^j \sum_{s=0}^{T}\left((1-c_1)(c_1)^s - (1-c_2)(c_2)^s\right)\left(\hat{Y}_{t+s} - \sigma\hat{C}_{t+s} - \frac{\tilde{\tau}_{t+s}^c}{1+\bar{\tau}^c} + \hat{\xi}_{t+s}\right) \\ &+ \tilde{E}_t^j \sum_{s=0}^{T}(1-c_1)(c_1)^s \hat{mc}_{t+s} + \tilde{E}_t^j \sum_{s=1}^{T}\left(\theta(c_1)^s - (\theta-1)(c_2)^s\right)\hat{\pi}_{t+s}\end{aligned} \tag{49}$$

with

$$c_1 = \omega\beta\bar{\Pi}^{\theta} \tag{50}$$

$$c_2 = \omega\beta\bar{\Pi}^{\theta-1} \tag{51}$$

Aggregating (49) yields

$$\int_0^1 \hat{d}_t(j)\mathrm{d}j = \bar{E}_t \sum_{s=0}^{T} \left((1-c_1)(c_1)^s - (1-c_2)(c_2)^s\right) \left(\hat{Y}_{t+s} - \sigma \hat{C}_{t+s} - \frac{\tilde{\tau}^c_{t+s}}{1+\bar{\tau^c}} + \hat{\xi}_{t+s}\right) \\ + \bar{E}_t \sum_{s=0}^{T} (1-c_1)(c_1)^s \hat{mc}_{t+s} + \bar{E}_t \sum_{s=1}^{T} \left(\theta (c_1)^s - (\theta-1)(c_2)^s\right) \hat{\pi}_{t+s} \quad (52)$$

Next, dividing by P_t, (18) can be written as

$$1 = \omega \Pi_t^{\theta-1} + (1-\omega) \int_0^1 d_t(j)^{1-\theta} \mathrm{d}j, \quad (53)$$

Log linearizing, this implies

$$\hat{\pi}_t = \frac{1-\omega \bar{\Pi}^{\theta-1}}{\omega \bar{\Pi}^{\theta-1}} \int_0^1 \hat{d}_t(j)\mathrm{d}j. \quad (54)$$

Plugging in in (52) gives

$$\hat{\pi}_t = \frac{1-\omega \bar{\Pi}^{\theta-1}}{\omega \bar{\Pi}^{\theta-1}} \Bigg[\bar{E}_t \sum_{s=0}^{T} \left((1-c_1)(c_1)^s - (1-c_2)(c_2)^s\right) \left(\hat{Y}_{t+s} - \sigma \hat{C}_{t+s} - \frac{\tilde{\tau}^c_{t+s}}{1+\bar{\tau^c}} + \hat{\xi}_{t+s}\right) \\ + \bar{E}_t \sum_{s=0}^{T} (1-c_1)(c_1)^s \hat{mc}_{t+s} + \bar{E}_t \sum_{s=1}^{T} \left(\theta (c_1)^s - (\theta-1)(c_2)^s\right) \hat{\pi}_{t+s} \Bigg] \quad (55)$$

B.3 Final equations

To complete the model, I first log-linearize the government budget constraint, (20), to

$$\tilde{b}_{t+1} = \frac{\bar{\Pi}}{\beta}\tilde{g}_t - \frac{\bar{w}\bar{s}\bar{\Pi}}{\beta}(\bar{\tau}^l(\hat{w}_t + \hat{H}_t) + \tilde{\tau}^l_t) - \frac{\bar{\Pi}}{\beta}(1-\bar{g})(\bar{\tau}^c \hat{C}_t + \tilde{\tau}^c_t) - \frac{\bar{\Pi}\bar{LS}}{\beta \bar{Y}}\hat{LS}_t + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{i}_t - \frac{1}{\beta}\hat{\pi}_t), \quad (56)$$

Next, we can log linearize the market clearing condition, (21)

$$\hat{Y}_t = (1-\bar{g})\hat{C}_t + \tilde{g}_t, \quad (57)$$

Next, I turn to aggregate labor

$$H_t = \int_0^1 H_t(j)\mathrm{d}j = \int_0^1 Y_t(j)\mathrm{d}j = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} \mathrm{d}jY_t = s_tY_t, \tag{58}$$

where $s_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} \mathrm{d}j$ is price dispersion in the economy in period t. Linearizing this equation and aggregating (42) wages and marginal costs can be written as

$$\begin{aligned} \hat{mc}_t = \hat{w}_t &= \eta\hat{H}_t + \sigma\hat{C}_t + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l} \\ &= (\eta + \frac{\sigma}{1-\bar{g}})\hat{Y}_t - \sigma\frac{\tilde{g}_t}{1-\bar{g}} + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l} + \eta\hat{s}_t \end{aligned} \tag{59}$$

Because all prices in the economy where set at different dates by the Calvo mechanism, price dispersion can be written as

$$\begin{aligned} s_t &= (1-\omega)\int_0^1 \left(\frac{p^*_t(j)}{P_t}\right)^{-\theta} \mathrm{d}j + \omega(1-\omega)\int_0^1 \left(\frac{p^*_{t-1}(j)}{P_t}\right)^{-\theta} \mathrm{d}j + \omega^2(1-\omega)\int_0^1 \left(\frac{p^*_{t-2}(j)}{P_t}\right)^{-\theta} \mathrm{d}j + ... \\ &= (1-\omega)\sum_{i=0}^{\infty}\omega^i \int_0^1 \left(\frac{p^*_{t-i}(j)}{P_t}\right)^{-\theta} \mathrm{d}j \end{aligned} \tag{60}$$

We can therefore write price dispersion as

$$\begin{aligned} s_t &= (1-\omega)\int_0^1 \left(\frac{p^*_t(j)}{P_t}\right)^{-\theta} \mathrm{d}j + \omega\Pi_t^\theta(1-\omega)\sum_{i=0}^{\infty}\omega^i \int_0^1 \left(\frac{p^*_{t-1-i}(j)}{P_{t-1}}\right)^{-\theta} \mathrm{d}j \\ &= (1-\omega)\int_0^1 d_t(j)^{-\theta}\mathrm{d}j + \omega\Pi_t^\theta s_{t-1} \end{aligned} \tag{61}$$

Price dispersion is log linearized to

$$\hat{s}_t = -\theta(1-\omega\bar{\Pi}^\theta)\int_0^1 \hat{d}_t(j)\mathrm{d}j + \theta\omega\bar{\Pi}^\theta\hat{\pi}_t + \omega\bar{\Pi}^\theta\hat{s}_{t-1} \tag{62}$$

Using (54), this can be written as

$$\hat{s}_t = \frac{\theta\omega\bar{\Pi}^{\theta-1}}{1-\omega\bar{\Pi}^{\theta-1}}(\bar{\Pi}-1)\hat{\pi}_t + \omega\bar{\Pi}^{\theta}\hat{s}_{t-1} \tag{63}$$

Finally, we can write real aggregate firm profits as

$$\Xi_t = \int_0^1 \Xi_t(j)\mathrm{d}j = \int_0^1 Y_t(j)\frac{P_t(j)}{P_t} - mc_t Y_t(j)\mathrm{d}j = (1 - mc_t s_t)Y_t, \tag{64}$$

which can be log-linearized to

$$\hat{\Xi}_t = \hat{Y}_t - \frac{\bar{s}\bar{w}}{1-\bar{s}\bar{w}}\left(\hat{mc}_t + \hat{s}_t\right). \tag{65}$$

Using (21) in (46) results in an expression for aggregate output.

$$\begin{aligned}
\hat{Y}_t &= \frac{1}{\rho}\tilde{b}_t + g_t + \delta\sum_{s=0}^{T}\beta^s((1-\bar{\tau}^l)\bar{E}_t\hat{w}_{t+s} - \bar{E}_t\tilde{\tau}^l_{t+s}) + \frac{(1-\bar{w}\bar{s})\bar{\Pi}}{\rho}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{\Xi}_{t+s}) \qquad (66)\\
&- \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{LS}_{t+s}) - \mu\sum_{s=1}^{T}\beta^s\sum_{j=0}^{s-1}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) + \frac{\bar{b}}{\rho}\sum_{s=0}^{T}\beta^s(\beta\bar{E}_t\hat{i}_{t+s} - \bar{E}_t\hat{\pi}_{t+s})\\
&- \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho}\sum_{j=0}^{T-1}(\bar{E}_t i_{t+j} - \bar{E}_t\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho}\bar{E}_t i_{t+T}\\
&\mu_\xi\xi_t - \mu\sum_{s=1}^{T}\beta^s\bar{E}_t\xi_{t+s} - \mu_c\tilde{\tau}^c_t - \frac{\bar{\Pi}(1-\bar{g})}{\rho}\left(1-\frac{1}{\sigma}\right)\sum_{s=1}^{T}\beta^s\bar{E}_t\tilde{\tau}^c_{t+s}
\end{aligned}$$

$$\delta = \frac{\bar{w}\bar{s}\bar{\Pi}}{\rho}\frac{\eta+1}{\eta} \tag{67}$$

$$\mu = \frac{\bar{\Pi}}{\rho}\left(\frac{\bar{w}\bar{s}}{\eta}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)\frac{1-\bar{g}}{\sigma}\right) \tag{68}$$

$$\mu_\xi = \frac{\bar{\Pi}}{\rho}\left(\frac{\beta-\beta^{T+1}}{1-\beta}\frac{\bar{w}\bar{s}}{\eta}(1-\bar{\tau}^l) + (1+\bar{\tau}^c)\frac{(1-\bar{g})\beta}{\sigma(1-\beta)}\right) \tag{69}$$

$$\mu_c = \frac{\bar{\Pi}}{\rho}\left(\frac{\beta-\beta^{T+1}}{1-\beta}\frac{\bar{w}\bar{s}}{\eta}\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c}+\frac{(1-\bar{g})\beta}{\sigma(1-\beta)}+(1-\bar{g})\right) \tag{70}$$

$$\rho = \frac{1}{1-\bar{g}}\left[\frac{\beta^{T+1}}{1-\beta}u_b+\left(\frac{\sigma}{\eta}\bar{w}\bar{s}(1-\bar{\tau}^l)+(1+\bar{\tau}^c)(1-\bar{g})\right)\bar{\Pi}\frac{1-\beta^{T+1}}{1-\beta}\right] \tag{71}$$

I now assume that agents know, or have learned about the above relations between aggregate variables (which hold in every period). Therefore, expectations about wages and profits can be substituted for, using (59) and (65). This gives the following system of 3 equations that, together with a specification of monetary and fiscal policy and price dispersion, completely describe our model

$$\begin{aligned}(1-\nu_y)\hat{Y}_t = {} & \frac{1}{\rho}\tilde{b}_t + g_t + \nu_\tau\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{\tau}^l_{t+s}) + \nu_g\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{g}_{t+s}) + \nu_y\sum_{s=1}^{T}\beta^s(\bar{E}_t\hat{Y}_{t+s}) \\ & + \nu_s\sum_{j=0}^{T}\beta^j(\bar{E}_t\hat{s}_{t+j}) - \mu\sum_{s=1}^{T}\beta^s\sum_{j=1}^{s}(\bar{E}_t i_{t+j-1}-\bar{E}_t\hat{\pi}_{t+j}) + \frac{\bar{b}}{\rho}\sum_{s=0}^{T}\beta^s(\beta\bar{E}_t\hat{i}_{t+s}-\bar{E}_t\hat{\pi}_{t+s}) \\ & - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho}\sum_{j=0}^{T-1}(\bar{E}_t i_{t+j}-\bar{E}_t\hat{\pi}_{t+j+1}) - \frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho}\bar{E}_t i_{t+T} - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{LS}_{t+s}) \\ & \mu_\xi\xi_t - \mu\sum_{s=1}^{T}\beta^s\bar{E}_t\xi_{t+s} + \nu_{c1}\tilde{\tau}^c_t + \nu_{c2}\sum_{s=1}^{T}\beta^s\bar{E}_t\tilde{\tau}^c_{t+s}\end{aligned} \tag{72}$$

$$\begin{aligned}\hat{\pi}_t = {} & \bar{E}_t\sum_{s=0}^{T}\left(\kappa_{y1}(c_1)^s+\kappa_{y2}(c_2)^s\right)\hat{Y}_{t+s} + \kappa_g\bar{E}_t\sum_{s=0}^{T}(c_2)^s\tilde{g}_{t+s} + \kappa_s\bar{E}_t\sum_{s=0}^{T}(c_1)^s\hat{s}_{t+s} \\ & + \kappa_c\bar{E}_t\sum_{s=0}^{T}(c_2)^s\tilde{\tau}^c_{t+s} + \kappa_\tau\bar{E}_t\sum_{s=0}^{T}(c_1)^s\tilde{\tau}^l_{t+s} + \bar{E}_t\sum_{s=1}^{T}\left(\kappa_{\pi1}(c_1)^s+\kappa_{\pi2}(c_2)^s\right)\hat{\pi}_{t+s} \\ & + \bar{E}_t\sum_{s=0}^{T}\left(\kappa_{\xi1}(c_1)^s+\kappa_{\xi2}(c_2)^s\right)\hat{\xi}_{t+s}\end{aligned} \tag{73}$$

$$\tilde{b}_{t+1} = \frac{\bar{\Pi}}{\beta}\tilde{g}_t - \frac{\bar{\Pi}}{\beta}\bar{\tau}^c(\hat{Y}_t - \tilde{g}_t) - \frac{\bar{\Pi}}{\beta}(1-\bar{g})\tilde{\tau}^c_t + \frac{1}{\beta}\tilde{b}_t + \bar{b}(\hat{i}_t - \frac{1}{\beta}\hat{\pi}_t) - \frac{\bar{\Pi}\bar{LS}}{\beta\bar{Y}}\hat{LS}_t$$
$$- \frac{\bar{w}\bar{s}\bar{\Pi}}{\beta}\left[\bar{\tau}^l\left((1+\eta+\frac{\sigma}{1-\bar{g}})\hat{Y}_t - \sigma\frac{\tilde{g}_t}{1-\bar{g}} + \frac{\tilde{\tau}^c_t}{1+\bar{\tau}^c} + \frac{\tilde{\tau}^l_t}{1-\bar{\tau}^l} + (1+\eta)\hat{s}_t\right) + \tilde{\tau}^l_t\right], \tag{74}$$

with

$$\nu_y = \frac{(1-\bar{s}\bar{w})\bar{\Pi}}{\rho} + \left(\delta(1-\bar{\tau}^l) - \frac{\bar{s}\bar{w}\bar{\Pi}}{\rho}\right)(\eta + \frac{\sigma}{1-\bar{g}}), \tag{75}$$

$$\nu_g = \left(\frac{\bar{s}\bar{w}\bar{\Pi}}{\rho} - \delta(1-\bar{\tau}^l)\right)\frac{\sigma}{1-\bar{g}}, \tag{76}$$

$$\nu_\tau = -\frac{\bar{s}\bar{w}\bar{\Pi}}{\rho(1-\bar{\tau}^l)}, \tag{77}$$

$$\nu_s = -\frac{\bar{s}\bar{w}\bar{\Pi}}{\rho}(\eta+1)\bar{\tau}^l, \tag{78}$$

$$\nu_{c1} = \delta\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} - \frac{\bar{s}\bar{w}\bar{\Pi}}{\rho(1+\bar{\tau}^c)} - \mu_c, \tag{79}$$

$$\nu_{c2} = \delta\frac{1-\bar{\tau}^l}{1+\bar{\tau}^c} - \frac{\bar{s}\bar{w}\bar{\Pi}}{\rho(1+\bar{\tau}^c)} - \frac{\bar{\Pi}(1-\bar{g})}{\rho}\left(1-\frac{1}{\sigma}\right), \tag{80}$$

$$\kappa_{y1} = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1+\eta)(1-\omega\beta\bar{\Pi}^{\theta}) \tag{81}$$

$$\kappa_{y2} = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1-\frac{\sigma}{1-\bar{g}})(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{82}$$

$$\kappa_g = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\frac{\sigma}{(1-\bar{g})}(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{83}$$

$$\kappa_s = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\eta(1-\omega\beta\bar{\Pi}^{\theta}) \tag{84}$$

$$\kappa_c = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\frac{1}{(1+\bar{\tau}^c)}(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{85}$$

$$\kappa_\tau = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\frac{1}{(1-\bar{\tau})}(1-\omega\beta\bar{\Pi}^{\theta}) \tag{86}$$

$$\kappa_{\pi 1} = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\theta \tag{87}$$

$$\kappa_{\pi 2} = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(\theta-1) \tag{88}$$

$$\kappa_{\xi 1} = \frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1-\omega\beta\bar{\Pi}^{\theta}) \tag{89}$$

$$\kappa_{\xi 2} = -\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}(1-\omega\beta\bar{\Pi}^{\theta-1}) \tag{90}$$

Linearized monetary and fiscal policy equations are given by

$$\hat{i}_t = \phi_1\hat{\pi}_t + \phi_2\hat{Y}_t, \tag{91}$$

$$\hat{LS}_t = \gamma_{LS}\tilde{b}_t. \tag{92}$$

C Model under infinite planning horizons

C.1 Output

When we let the planning horizon, T, go to infinity (66) can be written as

$$\hat{Y}_t = \frac{1}{\rho}\tilde{b}_t + g_t + \delta \sum_{s=0}^{\infty} \beta^s ((1-\bar{\tau}^l)((1-\alpha)E_t^F \hat{w}_{t+s} + \alpha E_t^b \hat{w}_{t+s}) - (1-\alpha)E_t^F \tilde{\tau}_{t+s}^l - \alpha E_t^b \tilde{\tau}_{t+s}^l) +$$

$$\frac{(1-\bar{w}\bar{s})\bar{\Pi}}{\rho} \sum_{s=0}^{\infty} \beta^s ((1-\alpha)E_t^F \hat{\Xi}_{t+s} + \alpha E_t^b \hat{\Xi}_{t+s}) - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho} \sum_{s=0}^{T} \beta^s ((1-\alpha)E_t^F \hat{LS}_{t+s} + \alpha E_t^b \hat{LS}_{t+s})$$

$$+ \frac{\mu\beta}{1-\beta}\xi_t - \mu \sum_{s=1}^{\infty} \beta^s ((1-\alpha)E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s})$$

$$- \mu_c \tilde{\tau}_t^c + \left(\frac{1}{\sigma} - 1\right) \frac{1-\bar{g}}{\rho}\bar{\Pi} \sum_{s=1}^{\infty} \beta^s ((1-\alpha)E_t^F \tilde{\tau}_{t+s}^c + \alpha E_t^b \tilde{\tau}_{t+s}^c)$$

$$- \frac{\mu\beta}{1-\beta} \sum_{s=0}^{\infty} \beta^s (((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s+1} + \alpha E_t^b \hat{\pi}_{t+s+1}))$$

$$+ \frac{\bar{b}}{\rho} \sum_{s=0}^{\infty} \beta^s (\beta((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s} + \alpha E_t^b \hat{\pi}_{t+s}))$$

Leading this equation 1 period and taking forward looking expectations gives

$$E_t^F \hat{Y}_{t+1} = \delta \sum_{s=1}^{\infty} \beta^{s-1} ((1-\bar{\tau}^l)((1-\alpha)E_t^F \hat{w}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{w}_{t+s}) - (1-\alpha)E_t^F \tilde{\tau}_{t+s}^l - \alpha E_t^F E_{t+1}^b \tilde{\tau}_{t+s}^l)$$

$$+ \frac{(1-\bar{w}\bar{s})\bar{\Pi}}{\rho} \sum_{s=1}^{\infty} \beta^{s-1} ((1-\alpha)E_t^F \hat{\Xi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\Xi}_{t+s}) - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho} \sum_{s=0}^{T} \beta^s ((1-\alpha)E_t^F \hat{LS}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{LS}_{t+s})$$

$$+ E_t^F g_{t+1} + \frac{1}{\rho}\tilde{b}_{t+1} + \frac{\mu\beta}{1-\beta} E_t^F \hat{\xi}_{t+1} - \mu \sum_{s=2}^{\infty} \beta^{s-1} ((1-\alpha)E_t^F \hat{\xi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\xi}_{t+s})$$

$$- \mu_c E_t^F \tilde{\tau}_{t+1}^c + \left(\frac{1}{\sigma} - 1\right) \frac{1-\bar{g}}{\rho}\bar{\Pi} \sum_{s=2}^{\infty} \beta^{s-1} ((1-\alpha)E_t^F \tilde{\tau}_{t+s}^c + \alpha E_t^F E_{t+1}^b \tilde{\tau}_{t+s}^c)$$

$$- \frac{\mu\beta}{1-\beta} \sum_{s=1}^{\infty} \beta^{s-1} (((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s+1} + \alpha E_t^F E_{t+1}^b \hat{\pi}_{t+s+1}))$$

$$+ \frac{\bar{b}}{\rho} \sum_{s=1}^{\infty} \beta^{s-1} (\beta((1-\alpha)E_t^F \hat{i}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{i}_{t+s}) - ((1-\alpha)E_t^F \hat{\pi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\pi}_{t+s}))$$

We can therefore write

$$(1-z_1\nu_y)\hat{Y}_t = (\beta-\alpha\beta\nu_y)E_t^f\hat{Y}_{t+1} + z_2\nu_y\hat{Y}_{t-1} + \frac{1}{\rho}(\tilde{b}_t - \beta\tilde{b}_{t+1}) + (1+z_1\nu_g)\tilde{g}_t - (\beta+\alpha\beta\nu_g)E_t^f\tilde{g}_{t+1}$$

$$+ z_2\nu_g\tilde{g}_{t-1} + \nu_\tau(z_1\tilde{\tau}_t^l + z_2\tilde{\tau}_{t-1}^l - \alpha\beta E_t^f\tilde{\tau}_{t+1}^l) + \nu_s(z_1\hat{s}_t + z_2\hat{s}_{t-1} - \alpha\beta E_t^f\hat{s}_{t+1})$$
$$+ \frac{\mu\beta}{1-\beta}\hat{\xi}_t - \left(\beta\frac{\mu\beta}{1-\beta} + \mu\beta(1-\alpha)\right)E_t^f\hat{\xi}_{t+1} - \frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}(z_1\hat{LS}_t + z_2\hat{LS}_{t-1} - \alpha\beta E_t^f\hat{LS}_{t+1})$$
$$+ \left(\nu_{c1} - \nu_{c2}\alpha\frac{\beta^2 d^2}{1-\beta d}\right)\tilde{\tau}_t^c + \nu_{c2}z_2\tilde{\tau}_{t-1}^c + (-\beta\nu_{c1} + \beta(1-\alpha)\nu_{c2})E_t^f\tilde{\tau}_{t+1}^c$$
$$+ \frac{\mu\beta}{1-\beta}((1-\alpha)E_t^F\hat{\pi}_{t+1} + \frac{\alpha d^2}{1-\beta d}\hat{\pi}_{t-1} - \frac{\alpha\beta d^2}{1-\beta d}\hat{\pi}_t)$$
$$- \frac{\bar{b}}{\rho}(z_1\hat{\pi}_t + z_2\hat{\pi}_{t-1} - \alpha\beta E_t^f\hat{\pi}_{t+1}) + (\frac{\bar{b}\beta}{\rho} - \frac{\mu\beta}{1-\beta})(z_1\hat{i}_t + z_2\hat{i}_{t-1} - \alpha\beta E_t^f\hat{i}_{t+1}), \tag{93}$$

where I follow the assumption that all agents can observe contemporaneous variables when making their decision (but not yet when forming expectations), and the assumption that backward-looking agents do not anticipate future shocks.

C.2 Inflation

When T goes to infinity, (73) can be written as

$$\hat{\pi}_t = \psi_t + \kappa_{y1}\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{Y}_{t+s} + \alpha E_t^b\hat{Y}_{t+s}) \tag{94}$$
$$\kappa_s\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{s}_{t+s} + \alpha E_t^b\hat{s}_{t+s}) + \kappa_\tau\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\tilde{\tau}_{t+s} + \alpha E_t^b\tilde{\tau}_{t+s})$$
$$+ \kappa_{\pi1}\sum_{s=1}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{\pi}_{t+s} + \alpha E_t^b\hat{\pi}_{t+s}) + \kappa_{\xi1}\sum_{s=0}^{\infty}(c_1)^s((1-\alpha)E_t^F\hat{\xi}_{t+s} + \alpha E_t^b\hat{\xi}_{t+s})$$

with

$$\psi_t = \kappa_{y2} \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \hat{Y}_{t+s} + \alpha E_t^b \hat{Y}_{t+s}) + \kappa_g \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \tilde{g}_{t+s} + \alpha E_t^b \tilde{g}_{t+s}) \tag{95}$$

$$+ \kappa_c \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \tilde{\tau}_{t+s}^c + \alpha E_t^b \tilde{\tau}_{t+s}^c) + \kappa_{\xi 2} \sum_{s=0}^{\infty} (c_2)^s ((1-\alpha) E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s})$$

$$+ \kappa_{\pi 2} \sum_{s=1}^{\infty} (c_2)^s ((1-\alpha) E_t^F \hat{\pi}_{t+s} + \alpha E_t^b \hat{\pi}_{t+s})$$

Writing one period ahead an taking forward-looking expectations gives

$$E_t^F \hat{\pi}_{t+1} = E_t^F \psi_{t+1} + \kappa_{y1} \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{Y}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{Y}_{t+s}) \tag{96}$$

$$+ \kappa_s \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{s}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{s}_{t+s}) + \kappa_\tau \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \tilde{\tau}_{t+s} + \alpha E_t^F E_{t+1}^b \tilde{\tau}_{t+s})$$

$$+ \frac{\kappa_{\pi 1}}{1-c_1} \sum_{s=2}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{\pi}_{t+s} + \alpha E_t^F E_{t+1}^b \hat{\pi}_{t+s}) + \kappa_{\xi 1} \sum_{s=1}^{\infty} (c_1)^{s-1} ((1-\alpha) E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s}),$$

$$E_t^F \psi_{t+1} = \kappa_{y2} \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \hat{Y}_{t+s} + \alpha E_t^b \hat{Y}_{t+s}) + \kappa_g \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \tilde{g}_{t+s} + \alpha E_t^b \tilde{g}_{t+s})$$

$$+ \kappa_c \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \tilde{\tau}_{t+s}^c + \alpha E_t^b \tilde{\tau}_{t+s}^c) + \kappa_{\xi 2} \sum_{s=1}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \hat{\xi}_{t+s} + \alpha E_t^b \hat{\xi}_{t+s})$$

$$+ \frac{\kappa_{\pi 2}}{1-c_2} \sum_{s=2}^{\infty} (c_2)^{s-1} ((1-\alpha) E_t^F \hat{\pi}_{t+s} + \alpha E_t^b \hat{\pi}_{t+s}) \tag{97}$$

Plugging in expectations of backward-looking agents we can write

$$\begin{aligned}\hat{\pi}_t =& c_1 E_t^F \hat{\pi}_{t+1} + \psi_t - c_1 E_t^F \psi_{t+1} + \kappa_{y1}(z_3 \hat{Y}_t + z_4 \hat{Y}_{t-1} - \alpha c_1 E_t^f \hat{Y}_{t+1}) \\ & \kappa_s(z_3 \hat{s}_t + z_4 \hat{s}_{t-1} - \alpha c_1 E_t^f \hat{s}_{t+1}) + \kappa_\tau(z_3 \tilde{\tau}_t^l + z_4 \tilde{\tau}_{t-1}^l - \alpha c_1 E_t^f \tilde{\tau}_{t+1}^l) \\ & + c_1 \kappa_{\pi 1}((1-\alpha) E_t^F \hat{\pi}_{t+1} + \frac{\alpha d^2}{1 - c_1 d} \hat{\pi}_{t-1} - \frac{\alpha c_1 d^2}{1 - c_1 d} \hat{\pi}_t) + \kappa_{\xi 1}(\hat{\xi}_t - \alpha c_1 E_t^f \hat{\xi}_{t+1})\end{aligned} \tag{98}$$

With,

$$\begin{aligned}\psi_t =& c_2 E_t^F \psi_{t+1} + \kappa_{y2}(z_5 \hat{Y}_t + z_6 \hat{Y}_{t-1} - \alpha c_2 E_t^f \hat{Y}_{t+1}) \\ & + \kappa_g(z_5 \tilde{g}_t + z_6 \tilde{g}_{t-1} - \alpha c_2 E_t^f \tilde{g}_{t+1}) + \kappa_c(z_5 \tilde{\tau}_t^c + z_6 \tilde{\tau}_{t-1}^c - \alpha c_2 E_t^f \tilde{\tau}_{t+1}^c) \\ & + c_2 \kappa_{\pi 2}((1-\alpha) E_t^F \hat{\pi}_{t+1} + \frac{\alpha d^2}{1 - c_2 d} \hat{\pi}_{t-1} - \frac{\alpha c_2 d^2}{1 - c_2 d} \hat{\pi}_t) + \kappa_{\xi 2}(\hat{\xi}_t - \alpha c_2 E_t^f \hat{\xi}_{t+1}),\end{aligned} \tag{99}$$

$$z_1 = 1 - \frac{\alpha \beta^2 d^2}{1 - \beta d} \tag{100}$$

$$z_2 = \frac{\alpha \beta d^2}{1 - \beta d} \tag{101}$$

$$z_3 = 1 - \frac{\alpha c_1^2 d^2}{1 - c_1 d} \tag{102}$$

$$z_4 = \frac{\alpha c_1 d^2}{1 - c_1 d} \tag{103}$$

$$z_5 = 1 - \frac{\alpha c_2^2 d^2}{1 - c_2 d} \tag{104}$$

$$z_6 = \frac{\alpha c_2 d^2}{1 - c_2 d} \tag{105}$$

D Time varying value functions

When the firm value function is given by (28), log linearizing an updated version of (48) will give extra terms corresponding to the time variation of the parameters of the value function. Inflation equation (55) then becomes

$$\begin{aligned}\hat{\pi}_t =&\frac{1-\omega\bar{\Pi}^{\theta-1}}{\omega\bar{\Pi}^{\theta-1}}\Bigg[\bar{E}_t\sum_{s=0}^{T}((1-c_1)(c_1)^s-(1-c_2)(c_2)^s)\left(\hat{Y}_{t+s}-\sigma\hat{C}_{t+s}-\frac{\tilde{\tau}^c_{t+s}}{1+\bar{\tau^c}}+\hat{\xi}_{t+s}\right)\\&+\bar{E}_t\sum_{s=0}^{T}(1-c_1)(c_1)^s\hat{mc}_{t+s}+\bar{E}_t\sum_{s=1}^{T}\left(\theta(c_1)^s-(\theta-1)(c_2)^s\right)\hat{\pi}_{t+s}\\&+\bar{E}_t\sum_{s=1}^{T}\left(\frac{\theta(c_1)^{T+1}}{1-c_1}-\frac{(\theta-1)(c_2)^{T+1}}{1-c_2}\right)\hat{\pi}^{LR}_t+\left((c_1)^{T+1}-(c_2)^{T+1}\right)\left(\hat{Y}^{LR}_t+\hat{\lambda}^{LR}_t\right)+(c_1)^{T+1}\hat{mc}^{LR}_t\Bigg]\end{aligned} \tag{106}$$

Similarly, making Λ and long run inflation in the household function time varying results in two extra terms in Equation (41) which now becomes

$$\begin{aligned}\tilde{b}^i_{t+T+1}=&\frac{u_b}{1-\beta}\hat{C}^i_{t+T}+\frac{u_b}{(1-\beta)\sigma}\frac{\tilde{\tau}^c_{t+T}}{1+\bar{\tau}^c}+\frac{u_b}{(1-\beta)\sigma}E^i_t i_{t+T}-\frac{u_b}{(1-\beta)\sigma}\xi_{t+T},\\&-\frac{u_b\bar{\Lambda}}{(1-\beta)\bar{C}(1+\bar{\tau}^c)}E^i_t\hat{\Lambda}^i_t+(\bar{b}-\frac{u_b}{(1-\beta)\sigma})E^i_t\hat{\pi}^{LR}_t\end{aligned} \tag{107}$$

As a result, two additional terms arise at the end of the output equation (72):

$$\begin{aligned}(1-\nu_y)\hat{Y}_t=&\frac{1}{\rho}\tilde{b}_t+g_t+\nu_\tau\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{\tau}^l_{t+s})+\nu_g\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{g}_{t+s})+\nu_y\sum_{s=1}^{T}\beta^s(\bar{E}_t\hat{Y}_{t+s})\\&+\nu_s\sum_{j=0}^{T}\beta^j(\bar{E}_t\hat{s}_{t+j})-\mu\sum_{s=1}^{T}\beta^s\sum_{j=1}^{s}(\bar{E}_t i_{t+j-1}-\bar{E}_t\hat{\pi}_{t+j})+\frac{\bar{b}}{\rho}\sum_{s=0}^{T}\beta^s(\beta\bar{E}_t\hat{i}_{t+s}-\bar{E}_t\hat{\pi}_{t+s})\\&-\frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho}\sum_{j=0}^{T-1}(\bar{E}_t i_{t+j}-\bar{E}_t\hat{\pi}_{t+j+1})-\frac{\beta^{T+1}}{1-\beta}\frac{u_b}{\sigma\rho}\bar{E}_t i_{t+T}-\frac{\bar{LS}\bar{\Pi}}{\bar{Y}\rho}\sum_{s=0}^{T}\beta^s(\bar{E}_t\hat{LS}_{t+s})\\&\mu_\xi\xi_t-\mu\sum_{s=1}^{T}\beta^s\bar{E}_t\xi_{t+s}+\nu_{c1}\tilde{\tau}^c_t+\nu_{c2}\sum_{s=1}^{T}\beta^s\bar{E}_t\tilde{\tau}^c_{t+s}+\frac{\beta^{T+1}}{1-\beta}\frac{u_b\bar{\Lambda}}{\bar{C}(1+\bar{\tau}^c)\rho}\hat{\Lambda}_t-\frac{\beta^{T+1}}{\rho}(\bar{b}-\frac{u_b}{(1-\beta)\sigma})\hat{\pi}^{LR}_t\end{aligned} \tag{108}$$