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Fully Funded Social Security Pensions, Lifetime Risk and Income

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Abstract

The paper analyzes the welfare consequences of insuring mortality risk by means of standard, fully funded Social Security pensions when individuals wish to make transfers to their heirs. In the presence of uninsured mortality risk, within-family transfers depend on realized lifespan. While crowding out private transfers, Social Security provides transfer insurance and insurance of the ex ante risk of future generations inheriting a particular amount of transfer wealth. We find that, once ex ante insurance is taken into account, Social Security is welfare improving over the long-run as long as capital is not too productive and the transfer motive is not too strong. Altruists gain far less from Social Security than egoists.

Keywords: Uninsured mortality risk, social security pensions, bequest motive, bequest insurance

JEL Classification: E6, D91, H55

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1 Introduction

There is a widespread use of Social Security pensions all around the world. Already in the late 1980's, the number of countries that ran some kind of old-age Social Security programme was 130, albeit of different size and coverage (see Sala-I-Martin, 1996). A common argument against Social Security pensions is that they reduce the national capital stock. Since capital is involved in the aggregate production process and its reduction will lead to less resources available in the future, Social Security pensions are seen as welfare-decreasing in the long-run, especially if financed on a pay-as-you-go[1] basis (Auerbach and Kotlikoff, 1987). Recent pension policies intend to bring in more funded elements into Social Security pensions. This paper investigates the welfare consequences of insuring mortality risk by means of fully funded Social Security. For example, the German system, the oldest formal Social Security pension system, started as a fully funded disability system in 1889, and was converted to a pay-as-you-go system only in the aftermath of the Great Depression and World War II. Chile is a prominent current example of a pension system with substantial reliance on funding. Countries like Japan and the Netherlands also have had fully funded pension systems for decades.

In aggregate, fully funded Social Security invests the contributions of a generation at the going interest rate and pays the proceeds plus the assets of those who die prematurely to the survivors of the *same* generation when old. Several empirical studies document a substantial reduction in bequest flows and capital (see, e.g., Auerbach et al., 1995). Two aspects of insurance that arise from premature death are of particular interest when individuals desire to make transfers to their heirs. First, Social Security provides intergenerational transfer insurance by smoothing transfers across states. Second, while death is certain, its timing is not, and those who die prematurely typically do so with some amount of wealth when they have made private savings to provide for their old age. Since the assets of the deceased are naturally transferred within their respective families, intragenerational inequality in wealth arises, so mortality among parents generates the ex ante risk of their children inheriting a particular amount of wealth. Due to the scheme's transfer insurance property, future generations face a more concentrated (or even degenerate) wealth distribution. Risk-averse individuals value the reduction in ex ante risk.

In this paper, capital reduction and insurance effects are combined in a simple dynamic overlapping generations model, wherein individuals live for at most two periods, facing a positive probability of dying just before reaching retirement in the second period. Intergenerational transfers occur *within* families, due to both premature death and individuals' desire to make positive inter-vivos transfers. While fully funded Social Security leads to a reduction in the long-run capital stock, we find that, once insurance is taken into account, the reduction of capital and long-run welfare gains are not mutually exclusive, whereby the reduction in capital is related to premature mortality. The point of departure is selfish behaviour, in the sense that bequest insurance provides

[1] If Social Security is financed on a pay-as-you-go basis, then the contributions of current workers finance pension benefits of current retirees. In contrast to the funded scheme studied here, contributions substitute for private savings without adding to the national capital stock.

no marginal utility, but the scheme promises a higher effective rate of return than the market.

The paper relates to the literature on the long-run consequences from annuitization by means of private markets (Kingston and Piggott, 1999 and Fehr and Habermann, 2008), and by means of Social Security (Hubbard and Judd, 1987, and most recently Caliendo et al., 2014). The main feature of this strand of the literature is the restriction to the life-cycle savings motive to finance old age consumption, coupled with a lump-sum redistribution of the assets of the deceased to all members of the next generation. The long-run welfare effect of Social Security pensions may be summarized in terms of two income effects. The first stems from a reduction of bequests, the second from the pension payment in old age. With a zero interest rate, both effects exactly cancel out in the long-run so that the introduction of a fully funded Social Security pension system leaves welfare unaffected in the long-run. Moreover, if the previous pension policy is revisable, then the pension system provides a higher effective rate of return than the market interest rate: The induced substitution effect is welfare improving in the long-run. This paper contributes to this literature by lifting the restriction to a lump-sum redistribution and studying the pension system's role in reducing the ex ante risks to lifetime income. We build on early works on within-family transfers in the absence of a bequest motive of Abel (1985) and Eckstein et al. (1985). To the author's best knowledge, only the appendix of Caliendo et al. (2014) provides a more recent treatment of within-family transfers in the pension context.[2] Our setup allows a generalization of Abel's (1985) model to the case where individuals have an operative bequest motive. In light of the very thin private annuities markets (see, e.g., Pashchenko, 2013) a thorough evaluation of Social Security should account for the system's capability to insure risks associated with premature mortality. However, previous studies with an operative bequest motive either assume that intragenerational heterogeneity does not arise (see, e.g., Sheshinski and Weiss, 1981 and Lockwood, 2012), or that private markets in annuities are available (see, e.g., Abel, 1986). We contribute to this literature by characterizing the long-run distribution of lifetime income in the presence of uninsured mortality risk and establishing allocative effects of the scheme relative to the world without it. We also quantify the ex ante welfare gain associated with the introduction of a fully funded Social Security pension system.

The paper is structured as follows: Section 2 presents the model setup. Section 3 establishes the welfare-dominance of Social Security over laissez-faire when individuals are perfectly selfish. The dynamics and long-run equilibria of the altruistic economy with and without Social Security are derived in Section 4. Rough calculations in Section 5 suggest that Social Security is also welfare improving over the long-run in an economy populated by altruistic individuals as long as capital is not too productive and the desire to make transfers to the heir is not too strong. A decomposition of the overall welfare effect into capital reduction and insurance gains is also provided. The main conclusions are summarized in Section 6.

[2]Bell and Gersbach (2013) study the impact of uninsured mortality risk on human capital formation. In their model, transfers are purely inter vivos, but individuals have two parents, which is an additional source of inequality.

2 The model economy

The demographic and economic environment is similar to the one used in Abel (1985), albeit the inclusion of an operative bequest motive requires a slighty more general description of the mortality process. Moreover, we have to distinguish transfers between the living (inter-vivos) from transfers that occur after death from the deceased to the living (bequest). We simply use the term transfer when both kinds of transfers are possible. Note that either the one or the other transfer occurs.

2.1 The demographic environment

The economy is populated in each period by two overlapping generations whose members live for at most two periods, namely, one young generation of workers and one old generation of retirees. There is a positive probability, q, of dying at the very end of the first period. At time $t = 0$, let there be a continuum of individuals indexed by $j \in [0, 1]$, all of whose parents survived to old age. All lineages stem from individuals born in $t = 0$. Each young individual gives birth (by parthenogenesis) to just one child prior to the realisation of the states premature death and survival. From then onwards, premature mortality sorts individuals into those whose parents survived both periods of life and those whose parents died prematurely. Let the state a member of lineage j is born into at any t be denoted by $I_t^j \in \mathcal{I} := \{0, 1\}$, where 0 and 1 indicate survival and premature death of the individual's parent, respectively. The lineage's particular mortality history up to and including period t is denoted by $\varsigma_t^j \in \mathcal{I}_0 \times \mathcal{I}_1 \times \cdots \times \mathcal{I}_t \equiv \mathcal{I}^t$. By assumption there is no aggregate uncertainty with regard to the number of premature deaths per generation.

2.2 The economic environment

There is a single commodity which can be either consumed or saved. Aggregate output, Y, is produced at the very beginning of each period by means of a production technology that is linear in both capital and labour. In per worker terms,

$$Y_t = F(1, K_t) = w + R \times K_t, \tag{1}$$

where K_t represents the aggregate capital stock at t. Capital has a lifetime of one period, so that K_t equals national savings in $t-1$. Note that (1) exhibits the property that capital and labour are perfect substitutes, with constant returns to scale. While the latter is standard in the literature, the former is very strong. Note however, that key features of the model, like the optimal size of the scheme, are unaffected by this assumption. Since factors are paid their respective marginal products, saving one unit of the good today yields R units in the next period. We say that capital is unproductive if $R = 1$. Due to linearity of the production function, factor prices are given and deterministic.

There is no private mechanism to pool individual mortality risk. In particular, private annuities markets are assumed to be absent[3]. The proceeds from savings of those who die prematurely are transferred to their respective direct descendants.

2.3 Preferences

All individuals have identical tastes. They derive utility from consumption in youth and old age, denoted by c_t^j and c_{t+1}^j, respectively. They also draw utility from inter-vivos transfers, $b_{t+1}^{j,0}$, and bequests in case of premature death, $b_{t+1}^{j,1}$, if these motives are operative. The superscripts 0 and 1 refer to the states survival I_t^0 and premature death I_0^1, respectively. Intergenerational transfers stem from own previous savings, a_t^j, and are transferred with interest.

Formally, j's preferences are represented by

$$U_t^j = u(c_t^j) + \beta(1-q)\left[u(c_{t+1}^{j,0}) + \nu(b_{t+1}^{j,0})\right] + \beta q \nu\left(b_{t+1}^{j,1}\right) \tag{2}$$

Pure impatience is captured by $0 < \beta \leq 1$.[4] Effective discounting is a mixture of impatience and mortality. The function ν represents the felicity yielded by giving. The felicity functions $u(\cdot)$ and $\nu(\cdot)$ are assumed to be strictly concave. Felicity from consumption in case of premature death is normalized to zero. Pathological behaviour is ruled out by the assumption $u(x) > \nu(x), \forall x$. The latter condition simply states that an individual who saved x/R units to old age prefers life to death even if the transfer motive is not strong enough to generate positive inter-vivos transfers. The transfer motive is operative if $\nu(\cdot) > 0$.

Note that the preferences differ conceptionally from 'pure' altruism à la Barro (1974) and Weil (1987) where individuals care about the utility of their offspring, in fact, the well-being of all future generations to come[5]. Here, transfers are motivated by the joy-of-giving, so that the individual draws utility directly from the transfers they make, pleasurable or when premature death afflicts them, but the pleasure does not depend on his children's utility gain (nor on the utility gain of any future generations of the same lineage that appear on the scene after their direct descendants). This kind of bequest motive is used by Abel (1986) and, more recently, Lockwood (2012). While this may look like selfish behaviour at first glance, the preferences defined above may be interpreted as the preferences of an individual who is concerned with the expected initial net wealth position of his offspring, where $\nu(\cdot)$ is the associated utility index.

Also note that the preferences defined in (2) implicitly impose risk neutrality with

[3]This assumption is also found in Sheshinski and Weiss (1981), Hubbard and Judd (1987) and Caliendo et al. (2014).

[4]Time preference is commonly measured as the marginal rate of substitution between young and old age consumption along a constant consumption path. With the time-additive preferences used here, β is just built in and invariant to consumption levels.

[5]This imposes a very strong assumption on the individual's capacity to form rational expectations about the decisions of succesive generations, and on the compatibility of their expectations with those of their heirs.

respect to lifetime risk[6].

2.4 Fully funded Social Security pensions

Under fully funded Social Security Pensions, contributions are proportional to wages and are invested at the going market interest rate and paid as pension benefits to the surviving members of the same generation one period later. The scheme pools the risks of premature mortality: the pension claims of those who die prematurely are divided among the survivors of the *same* generation. In the happy event that individual j, born in the beginning of period t, survives into old age, she will receive $a_t^j R$ from her privately placed savings a_t^j, and

$$\tau_t^j wR + \tau_t^j \frac{q}{1-q} wR$$

from the pension scheme, where τ_t^j denotes period-t contribution rate for individual j. Note that the second term captures the effect of pooling on old age income. The latter is

$$\left[a_t^j + \tau_t^j w/(1-q)\right] R. \tag{3}$$

If she dies prematurely, then her heir will receive only $a_t^j R$ as bequest. Note that private savings and Social Security pensions are *not* perfect substitutes: While saving one unit conventionally yields R units in the next period, whether the indiviudal survives or not, the scheme promises a higher rate of return on contributions, $R/(1-q)$, when the individual survives, and pays nothing when she dies prematurely. The pension claim is a state-dependent, i.e. risky, asset that can be used to equalize bequest streams across states. Also note that the scheme is actuarially fair, so that the expected return (including the death case) of the individual's portfolio is unaffected by the scheme. Since the scheme is fully funded, the contributions to the scheme add to the next period's capital stock, so $K_{t+1} = R \times \int_0^1 \left(a_t^j + \tau_t^j w\right) dj$.

We analyse two cases. First, individuals treat the pension policy as given. Second, each generation can freely decide on the contribution rate. In the latter case, we consider a string of consecutive generations, each member $j \in [0,1]$ of which chooses the contribution rate so as to maximize her expected utility. This assumption is plausible in a two-period setting.

[6] In fact, the preferences defined in (2) are a special case of

$$U_t^j = (1-q)\phi\left[u(c_{t+1}^j) + \nu(b_{t+1}^j)\right] + q\phi\left[\nu(w(1-\tau_t^j) + b_t^j)\right]$$

where the curvature of ϕ capture individual's lifetime risk aversion (see Kihlstrom and Mirman, 1974). Bommier (2013) shows that only an exponential form of ϕ is compatible with time-consistent preferences when individuals live for more than two periods. Since ϕ is assumed linear in this paper, (2) defines preferences of an individual who is risk neutral with respect to mortality risk.

3 The egoistic economy

This section studies the case of perfectly selfish individuals. Set $\nu(\cdot) = 0$ so that the proceeds from saving generate no marginal utility when the individual dies prematurely, be they transferred to the next generation or to the survivors of the same generation.[7] Obviously, there will be no intentional bequests. We establish global convergence to the unique long-run equilibrium. Whether the capital stock with the scheme falls short of, or exceeds, the laissez-faire capital stock in the long run depends on the intertemporal elasticiy of substitution. Two welfare results are derived: First, a fully funded Social Security scheme with time-invariant fixed contribution rate is welfare-increasing. Second, the time-consistent pension policy, to which all subsequent generations will agree, does even better. We show in Sections 4.1 and 4.2 that essential properties carry over to the case of altruistic individuals.

3.1 Laissez-faire and a fixed contribution rate scheme

As a preliminary to the welfare analysis, I establish existence of a unique stationary equilibrium if the contribution rate is treated as given when individuals make their consumption-savings decisions, i.e. $\tau_t = \tau$. We assume throughout that the fixed contribution rate is small enough to ensure that even the poorest individuals make positive private saving. Laissez-faire serves as the benchmark to evaluate the welfare consequences of Social Security. We use the term 'Laissez-faire' to describe the world without the scheme, so legal and administrative basis of the latter is absent. The laissez-faire economy corresponds to $\tau = 0$.

When young, an egoistic individual j, born in period t, decides on private savings and consumption in youth. When old, if she survives, she simply consumes the proceeds from savings. The maximization problem of individual j in the presence of the fixed contribution rate scheme,

$$\max_{a_t^j} u(w(1-\tau) + b_t^j - a_t^j) + \beta(1-q)u\left(a_t^j R + \frac{\tau w R}{1-q}\right), \tag{4}$$

yields the Euler equation

$$u'(c_t^j) = \beta(1-q)Ru'(c_{t+1}^j). \tag{5}$$

Note the presence of mortality in (5), whose form is such that the scheme with a fixed contribution rate does not affect intertemporal trade-offs. Let u be iso-elastic, with parameter σ, $u(c) = u_0 + \frac{c^{1-\sigma}-1}{1-\sigma}$, $u_0 > 0$. Then, the associated savings function is $a_t^j = \bar{a}_e(w(1-\tau) + b_t^j) - (1-\bar{a}_e)\tau w/(1-q)$, where $\bar{a}_e = \left[1 + (\beta(1-q)R^{1-\sigma})^{-1/\sigma}\right]^{-1}$ is the egoistic individual's propensity to save out of the present value of lifetime income. Note that $\tau \leq \tau^0 = \frac{\bar{a}_e(1-q)}{1-q\bar{a}_e}$ ensures non-negative private savings of those individuals

[7] Similar models are derived in Abel (1985) and Eckstein et al. (1985). I use the model for evaluating long-run welfare consequences of the scheme.

who do not receive any bequest. Plugging the optimal decision back into the direct utility function, gives indirect utility of the egoistic individual j, $V^j_{t,e}(\tau)$[8].

Since accidental bequests stem from the parent's savings,

$$a^j_t = \bar{b}^j_t a^j_{t-1} + \bar{w}(\tau), \tag{6}$$

where

$$\begin{aligned} \bar{w}(\tau) &= \bar{a}_e \times w(1-\tau) - (1-\bar{a}_e)\tau w/(1-q), \\ \bar{b}^j_t &= 0\chi(I^j_{t-1} = 0) + R\bar{a}_e \chi(I^j_{t-1} = 1), \\ E\bar{b}_t &= R\bar{a}_e q, \end{aligned}$$

and χ is an indicator function so that $\chi(z) = 1$ if z is true, and $\chi = 0$ if z is false. $I^j_{t-1} = 0$ indicates that the individual's parent survived into old age, and makes no intergenerational transfer. $I^j_{t-1} = 1$ indicates that the individual's parent died early and left an accidental bequest to the heir.

Lemma 1 establishes convergence to the unique stationary equilibrium. Part *(ii)* of the Lemma states that the fully funded pension scheme with fixed contribution rate crowds out the long-run aggregate capital stock by reducing the accidental bequests. With an eye on Propositions 1 and 2, note that in the current setup, aggregates coincide with ex ante expectations.

Lemma 1. ***Convergence to the unique stationary equilibrium distribution*** *Let $R\bar{a}_e q < 1$ and $\tau \leq \tau^0$. Then*

(i) a^j_t converges to a^j_∞ as $t \to \infty$, where the distribution of a^j_∞ is the unique solution to[9]

$$a^j_\infty =^d \bar{b}^j_t \times a^j_\infty + \bar{w}(\tau),$$

which is independent of a^j_0.

(ii) the long-run aggregate capital stock

$$Ea^j_t = K_e(\tau) = \bar{w}(\tau)/(1 - R\bar{a}_e q) \tag{7}$$

is strictly decreasing in the contribution rate τ.

Proof. (i) By iterating (6), one obtains

$$a^j_\infty =^d \bar{w}(\tau) + \bar{w}(\tau)\sum_{i=1}^{\infty} \bar{b}^j_1 \bar{b}^j_2 \cdots \bar{b}^j_{i-1}. \tag{8}$$

By Jensen's inequality, $E\bar{b}_t = R\bar{a}_e q < 1$ implies $E\log(\bar{b}^j_t)(\leq \log E\bar{b}^j_t) < 0$, which is a sufficient condition for convergence of the infinite sum (see Vervaat, 1979). (ii) $\bar{w}$ is strictly decreasing in τ. $\square$

[8] We use $V^j_{t,e}(\tau)$ as a shortcut for $V^j_{t,e}\left(c^j_t(\tau), c^j_{t+1}(\tau)\right)$, where $c^j_t(\tau)$ and $c^j_{t+1}(\tau)$ denote the optimal decision in the presence of the system with a fixed contribution rate.

[9] $=^d$ means equality in distribution.

In an economy populated by overlapping generations the reduction in the aggregate capital stock is a misleading indicator for the welfare consequences of the scheme. We next show that, once ex ante insurance is taken into account, the crowding out of capital is consistent with welfare improvements. First note that Lemma 1 implies convergences in distribution of all other relevant variables. In particular, lifetime income and therefore consumption in youth, $c_t^j(\tau) = (1-\bar{a}_e)\left(w + b_t^j + \frac{\tau w}{1-q}\right)$ converges in distribution.Since the fraction q of total population dies early and bequests are transferred with interest, i.e. $Eb_t^j = qRK_e(\tau)$, aggregate first-period consumption in the stationary equilibrium is given by

$$Ec_t^j(\tau) \;=\; C_e(\tau) = (1-\bar{a}_e)\left(\frac{w}{1-R\bar{a}_e q} + \frac{\tau w q(1-R)}{(1-q)(1-R\bar{a}_e q)}\right), \tag{9}$$

where the term in brackets is the ex ante expected value of the present value of lifetime income. The latter is unaffected by the scheme whenever capital is unproductive, and so is $Ec_t^j(\tau)$.

In the egoistic economy, different mortality histories across lineages result in different levels of lifetime income. As a consequence, individuals can be unambiguously identified by their lineages' mortality histories. Without ambiguity, therefore, I drop the index j and reinterpret, as Abel (1985) does, the time index t as the number of previous consecutive generations within a particular lineage whose members died prematurely. By the consumption Euler equation (5), $c_{t+1}^j(\tau)$ is proportional to $c_t^j(\tau)$. It is therefore enough to focus on first-period consumption. In order to prepare for Proposition 1, the following lemma restates a result provided by Abel (1985) in a slidely different form and also recognize that we work with a geometric distribution.

Lemma 2. ***(Abel, 1985) Geometric long-run distribution***

In the long-run, individuals are geometrically distributed with probability mass function $q^t(1-q)$. First-period consumption of a type-t individual is

$$c_t(\tau) \;=\; w(1-\tau) - \bar{w}(\tau)\left(\bar{a}_e R\right)^t + \bar{w}(\tau)(R-1)\sum_{i=0}^{t}\left(\bar{a}_e R\right)^i. \tag{10}$$

If $R\bar{a}_e q < 1$, then $\lim_{t\to\infty} c_t(\tau) = \infty$ is compatible with the stationary equilibrium defined in Lemma 1. In that case, there must be both winners and losers in the long run (most likely, earlier). Figure B.1 suggests that this true even when the long-run distributions have bounded supports. Suppose, for example, that capital is unproductive (this case is illustrated in Figure B.1(a)). Solving $c_t(\tau) \geq c_t(0)$ for t shows that welfare of all but those whose parents died prematurely and so left no bequests (the poorest without the scheme) is reduced relative to the laissez-faire, $\tau = 0$; for they experience a negative income effect due to a reduction in bequests that dominates the positive income effect stemming from the scheme. Figure B.1(b) illustrates this result when $R >> 1$. Since type-t individuals occur with frequency $q^t(1-q)$, individuals whose parents died prematurely make up the fraction $(1-q)$ of the total young population. Anticipating the discussion in Section 5, let the economically active years start at the age of 20 and let life expectancy at birth be 80 years (i.e. $q = 1/2$). In that case, the scheme makes one half of the young population better off. However, *ex post* inequality translates into *ex ante* uncertainty, and the scheme produces a more consentrated distribution of lifetime income in the stationary equilibrium. Risk-averse individuals value the reduction in ex ante risk.

The next proposition avoids distributional aspects by evaluating welfare on the basis of Rawls's (1973) veil of ignorance. It exploits the fact that, under (22), individuals consume a constant fraction of lifetime income for all $\sigma > 0$ and all $0 < \beta \leq 1$, thereby generalizing Caliendo et al.'s (2014) Proposition 5. Suppose *all* individuals are present behind the veil and are asked whether they prefer to be born into the laissez-faire economy or the economy that runs the fully funded pension scheme. Due to the direct link between an individual's family mortality history and wealth at birth in the egoistic economy, ignorance refers to the ex ante risk of being born into a particular lineage.

Proposition 1. *Ex ante welfare-improving Social Security pensions with a fixed contribution rate*

Suppose that capital is unproductive, with $q > 0$, and $\tau \leq \tau^0 = \frac{\bar{a}_e(1-q)}{1-q\bar{a}_e}$. Then, fully funded Social Security Pensions increase ex ante welfare.

Proof. If $R = 1$, then (9) and (10) imply $c_t(\tau) = w(1 - \bar{a}_e^{t+1}) + w\tau\left(\frac{1-\bar{a}_e q}{1-q}\bar{a}_e^t - 1\right)$, with $Ec_t(\tau) = C_e(\tau) = \frac{(1-\bar{a}_e)w}{1-R\bar{a}_e q}$. Therefore, $C_e(\tau) > c_0(\tau) > c_0(0)$, and $C_e(\tau) < c_t(\tau) < c_t(0)$, $t > 0$, implying that the scheme induces a mean-preserving reduction in the spread of consumption. The latter, in turn, implies second-order stochastic dominance, i.e. $\int_t V_t(\tau) > \int_t V_t(0)$, which confirms the claim. $\square$

Proposition 1 is important, because it illustrates that the welfare impact of fully funded Social Security pensions is systematically underestimated in the literature. In fact, there are several contributions in the literature on accidental bequests in egoistic economies that find no welfare impact stemming from the scheme if capital is unproductive (see e.g. Hubbard and Judd, 1987, and more recently Caliendo et al., 2014, among others). The underlying assumption in the literature is that accidental bequests are pooled and then transferred anonymously. In the long-run, the income effect from reduced accidental bequests and the income effect stemming from reduced net wage income and additional pension benefit exactly cancel out. Therefore, long-run welfare is unaffected by the scheme when capital is unproductive.

3.2 Time-consistent pension policy

This section establishes that the welfare gains from Social Security Pensions are further underestimated relative to those found in 3.1; for individuals treated the contribution rate as given when making their decisions. On the other extreme, contemporary generations are able to decide for themselves on the individual contributions made to the pension system. In fact, in the two-period setup presented here, each generation is able to revise previous pension policy without producing intergenerational conflicts. This implies that the pension system essentially acts like privately organised, actuarially fair annuities markets, with a coompetitive premium taken by the firms which fully and exclusively reflects the prevailing mortality regime.[10]

The maximization problem of individual j is

$$\max_{c_t^j, c_{t+1}^j, \tau_t^j} u(c_t^j) + \beta(1-q)u(c_{t+1}^j) \tag{11}$$

subject to

$$\begin{aligned} c_t^j &= w(1-\tau_t^j) + b_t^j - a_t^j & (12)\\ c_{t+1}^j &= a_t^j R + \tau_t^j wR/(1-q) & (13)\\ \tau_t^j &\in [0,1]. & (14) \end{aligned}$$

The first-order condition with respect to τ_t^j is

$$u'(c_t^j) = \beta R u'(c_{t+1}^j), \tag{15}$$

where mortality has dropped out. Since individuals do not value their wealth after death, the scheme promises a higher expected rate of return than the market does, and egoistic individuals do not save privately. Of course, optimality of full annuitization is a well-known result if individuals save out of pure selfishness (see Yaari, 1965). In the egoistic economy, full annuitization eliminates all bequests. Moreover, under the assumption that all are alike at $t = 0$, and receive zero bequests, there is intragenerational agreement on the optimal policy for all $t = 0, 1, 2, \ldots, \infty$. From (15) we have

Lemma 3. ***Unique time-consistent pension policy***

There exists a unique time-consistent optimal contribution rate:

$$\tau_e^* = \left[1 + (\beta R^{1-\sigma})^{-1/\sigma}/(1-q)\right]^{-1}, \ \forall j, t. \tag{16}$$

The pension policy (16) is termed time-consistent: All generations are able, but not willing, to revise previous policies. The time consistent policy is therefore sustainable in the long-run. The associated indirect utility is denoted by $V_t(\tau_e^*)$.

For the remainder of this section, let $\beta^{-1} = R = 1$. In that case, Abel (1985) shows that the long-run capital stock of the laissez-faire economy ($\tau = 0$) falls short of the one associated with the time-consistent policy in (16), i.e. $\tau_e^* w$ if and only if $\sigma < \widetilde{\sigma} \equiv \left[1 - \frac{\ln(1+q(1-q))}{\ln(1-q)}\right]^{-1} < 1$. The reason is that each generation can revise previous policy, so the increase in effective rate of return affects intertemporal trade-offs (compare (15) and (5)). The associated income effect induces the young individual to increase

[10]One can think of a large number of insurance companies which receive the market interest rate on their reserves and earn zero profits under perfect competition.

current consumption and to reduce savings. In contrast, the associated substitution effect induces the individual to save more to old age. An intertemporal elasticity of substitution larger than one ($\sigma < 1$) and perfectly selfish individuals imply that the substitution effect dominates the income effect, so the propensity to save is higher with the scheme. Whether this effect compensates for the elimination of accidental bequests depends on whether σ is small enough. The following corollary states that if the fixed contribution rate of Section 3.1 is small enough, then the associated capital stock exceeds the one associated with the time-consistent policy defined in (16).

Corollary 1. *Let $\beta^{-1} = R = 1$, and $\sigma > \widetilde{\sigma}$. Then,*

$$K_e(\tau) > K_e(\tau_e^*) \text{ as } \tau < \widetilde{\tau},$$

where $\widetilde{\tau} = \tau^0 - \frac{1-q}{1+\frac{1}{1-q}} > 0$.

As already shown in Section 3.1, the evolution of aggregates is an imperfect indicator of welfare. The next proposition states that the time-consistent policy is ex ante welfare-improving and dominates the system with fixed contribution rate for all $\sigma > 0$ and all $\tau < \tau^0$.

Proposition 2. ***Dominance of time-consistent Policy***

If $R = 1$, then the time-consistent pension policy defined in Lemma 3 dominates, in welfare terms, the fully funded scheme with fixed contribution rate $\tau < \tau^0$.

Proof. The proof is accomplished by constructing an auxiliary consumption allocation induced by a policy that redistributes the assets of those who die prematurely to the next generation in a lump-sum manner. With this policy, all members of a generation receive identical accidental bequests, and the Euler equation is of the form of (5). The associated first-period consumption is $c_t^{LS} = (1 - \bar{a}_e)\left(w + b^{LS}\right)$, where $b^{LS} = \frac{qR}{1+(\beta(1-q)R^{1-\sigma})^{-1/\sigma} - qR} w$. First, $R = 1$ implies $Ec_t^{LS} = Ec_t(\tau)$ and therefore $V^{LS} > V(\tau)$; for the lump-sum policy induces a mean-preserving elimination of any spread in consumption. Second, if $R = 1$, then (c_t^{LS}, c_{t+1}^{LS}) and $(c_t(\tau_e^*), c_{t+1}(\tau_e^*))$ are on the same budget line. Therefore, while feasible, the auxiliary allocation is not chosen. By revealed preference, $V(\tau_e^*) > V^{LS} > V(\tau)$. □

The conclusion from Propositions 1 and 2 is that the crowding out of bequests and capital induced by fully funded schemes is consistent with long-run welfare gains whenever capital is not too productive, a claim that follows at once from the strict inequalities above and the continuity of V. Moreover, if each generation is able to revise previous policy, then the scheme even increases long-run capital stock for plausible values of the intertemporal elasticity of substitution. The associated substitution effect is positive in welfare terms. From a generational perspective, the revised contribution rate setup seems plausible; for the scheme should be regarded as acting like annuities markets, although contributions are wage-related. As a final remark, note that neither of the policy regimes is Pareto-improving relative to the laissez-faire. This is obvious from the discussion following Lemma 2 for the case of a fixed contribution rate. Moreover, the time-consistent pension policy yields $c_t(\tau_e^*) = w/(2-q) < w = \lim_{t\to\infty} c_t(0)$.

4 The altruistic economy

This section studies the dynamics and long-run behaviour of the altruistic economy with and without Social Security. It uses the apparatus of Section 3 to establish convergence to the unique stationary equilibrium in the presence of uninsured mortality risk and an operative bequest motive. The clear-cut welfare results derived for the egoistic economy studied in the previous section are feasible because there are direct relationships, first, between an individual's family mortality history and her expected lifetime income, and second, between expected lifetime income and welfare. Both links break in the altruistic economy. The welfare consequences of the scheme are therefore studied numerically in Section 5.

4.1 Dynamics under laissez-faire

When young, individual j, born in period t, decides on private savings and consumption in youth. When old, if she survives, she allocates the proceeds from savings between old-age consumption and inter-vivos transfers. These choices are determined by the individuals' attitudes towards time, risk and their heirs.

The individual's maximization problem is

$$\max_{a_t^j} U_t^j$$

subject to

$$\begin{aligned} c_t^j &= w + b_t^j - a_t^j, & (17) \\ \left[c_{t+1}^{j,0}, b_{t+1}^{j,0}\right] &\equiv \arg\max \left[u(c_{t+1}^j) + \nu(b_{t+1}^j)\right] \quad \text{s.t. } c_{t+1}^j + b_{t+1}^j = a_t^j R & (18) \\ b_{t+1}^{j,1} &= a_t^j R & (19) \\ a_t^j &\geq 0, \; b_{t+1}^{j,0} \geq 0, \; c_{t+1}^{j,0} \geq 0. & (20) \end{aligned}$$

given the bequest b_t^j.[11] Intergenerational transfers, pleasurable or not, are non-negative; for by assumption, individuals cannot be forced to accept negative bequests (liabilities left by their parents). For simplicity, both kinds are distributed at the very beginning of each period. Since all individuals born in $t = 0$ have parents who survived to old-age, all receive the same bequests at that point in time so that there is no intragenerational heterogeneity at this point. Without loss of generality, let $b_0^j = 0 \, \forall j$. Unlike in the case of egoistic individuals, problem (2) - (20) needs to be solved by backwards induction. Assuming an interior solution to (18), $u'(c) = \nu'(b)$.

The first-order conditions with respect to a_t^j may be written as[12]

$$u'(c_t^j) \;\geq\; \beta(1-q)R\nu'(b_{t+1}^{j,0}) + \beta q \nu'(Ra_t^j)R. \qquad (21)$$

[11] By the assumption that bequests received are known prior to the decision, the individual is only concerned with the uncertainty associated with her own life.

[12] It is readily verified that the second derivative with respect to a_t^j is negative, so the solution is indeed a maximum.

Individuals save out of a pure life-cycle motive (i.e. for old-age consumption), and out of an altruistic motive (i.e. for making direct intergenerational transfers). Recalling (18), both motives are summarized in the first term on the right-hand side of (21). The last term arises due to the prevalence of uninsured mortality risk, which tends to increase private savings.

Let u and ν be iso-elastic, with common parameter σ:[13]

$$u(c) = u_0 + \frac{c^{1-\sigma}-1}{1-\sigma}, \tag{22}$$

$$\nu(b) = \gamma\frac{b^{1-\sigma}-1}{1-\sigma}. \tag{23}$$

The parameter $u_0 > 0$ determines the felicity gap between life and death, a condition which is implicit in the above requirement of non-pathological behaviour with respect to the bequest motive (see Section 2.3). Also note that $u_0 > 0$ does not affect the inter-temporal consumption allocation. To use terminology carefully, $\sigma > 0$ is, by definition, the elasticity of marginal felicity from consumption and intergenerational transfers, respectively. It is readily verified that $\lim_{\sigma\to 1} u(c) = \ln(c)$. The parameter $0 < \gamma \leq 1$ measures the strength of the bequest motive. Note that (23) implies that inter vivos transfers are normal goods.

Since (22) and (23) satisfy the lower Inada condition (that is, $\lim_{x\to 0} u'(x) = \infty$, and $\lim_{x\to 0} \nu'(x) = \infty$), inter vivos transfers are always positive if the parents survive into old age. The consumption-transfer allocation is

$$b_{t+1}^{j,0} = \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a}(w + b_t^j) \tag{24}$$

$$c_{t+1}^{0} = \frac{1}{1+\gamma^{1/\sigma}} R\bar{a}(w + b_t) \tag{25}$$

Moreover, individuals optimally save a constant fraction of the expected present value of their lifetime incomes, i.e. $a_t^j = \bar{a}(w + b_t^j)$, where

$$\bar{a} = \left\{1 + \left[\beta R^{1-\sigma}\left((1-q)\left(1+\gamma^{1/\sigma}\right)^{\sigma} + q\gamma\right)\right]^{-1/\sigma}\right\}^{-1} < 1 \tag{26}$$

denotes the propensity to save out of lifetime income under laissez-faire, a propensity which is increasing in the strength of the bequest motive. Note that lifetime income equals initial wealth only if previous pension policy can be revised. Note that the desire to make transfers to the heir increases the said propensity relative to the case of egoistic individuals, i.e. $\bar{a} > \bar{a}_e$.

In the absence of the scheme, the randomness of premature death causes individuals to differ in the bequests they receive at the start of the first period of their lives: b_t^j arises

[13] For simplicity, we assume that both kinds of transfers receive the same weight γ, albeit one can imagine that inter vivos may be higher valued by the individual. Abel (1986) and Lockwood (2012) employ these funtional forms with $u_0 = 0$. Lockwood (2012) extends (23) to cover inter-vivos transfer as luxury goods.

from the mortality history ς_t^j. Since premature death is identically and independently distributed, intragenerational heterogeneity with respect to b_t^j arises and is increasing as time passes. Recalling (19) and (24), savings evolve according to the stochastic first-order difference equation with an identically and independently distributed coefficient

$$a_t^j = \bar{b}_t^j a_{t-1}^j + \bar{w}, \tag{27}$$

with $\bar{w} = \bar{a} \times w$ and the random variable $\bar{b}_t^j = \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a}\chi(I_{t-1}^j = 0) + R\bar{a}\chi(I_{t-1}^j = 1)$. $I_{t-1}^j = 0$ indicates that the individual's parent survived into old age, and, in contrast to the previous section, makes an inter-vivos transfer. $I_{t-1}^j = 1$ indicates that the individual's parent died early and left an accidental bequest to the heir. Note that $E\bar{b}_t = q \times \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a} + (1-q) \times R\bar{a} = R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}}$.

The stationary equilibrium is given by prices (R, w), an allocation $(c_t^j, c_{t+1}^j, b_{t+1}^j, a_t^j)$, and a time-invariant distribution lifetime income such that, given (R, w), the allocation maximizes expected utility defined in (2), and individual choices are consistent with the economy-wide resource constraint. Lemma 4 generalizes Lemma 1 by establishing convergence of the altruistic economy to the unique stationary equilibrium.

Lemma 4. ***Convergence in distribution***

(i) Let $R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1$. Then, a_t^j converges to a_∞^j as $t \to \infty$, where the distribution of a_∞^j is the unique solution to

$$a_\infty^j =^d \bar{b}_t^j \times a_\infty^j + \bar{w},$$

which is independent of a_0^j.

(ii) If $R\bar{a} < 1$, then the long-run capital distribution has bounded support, $a^j \in [\bar{w}/(1-\gamma^{1/\sigma}\bar{a}R/(1+\gamma^{1/\sigma})), \bar{w}/(1-\bar{a}R)]$.

Proof. By iterating (27), one obtains

$$a_\infty^j =^d w + w\sum_{i=1}^{\infty} \bar{b}_1^j \bar{b}_2^j \cdots \bar{b}_{i-1}^j \tag{28}$$

(i) $E\bar{b}_t = R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1$ implies (by Jensen's inequality) $E\log(\bar{b}_t^j)(\leq \log E\bar{b}_t^j) < 0$, which is a sufficient condition for convergence of the infinite sum, (see Vervaat, 1979). (ii) The bounds follow by setting $\bar{b}_t = R\bar{a}, t = 0, 1, 2, \ldots, \infty$ and $\bar{b}_t = \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}}R\bar{a}$, $\frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1$, $t = 0, 1, 2, \ldots, \infty$, respectively. □

The long-run distribution deserves some comment. First, if capital is unproductive (i.e. $R = 1$), then the support is bounded without further assumptions. From (26), however, the bounds are growing with R whenever $\sigma < 1$. If $R > 1$, then convergence and boundedness require the interest rate to be sufficiently small, albeit $R >> 1$.

Intuitively, since bequests are transferred with interest, too large an interest rate makes bequests grow without bounds as time passes, which is at odds with the requirement of a stationary equilibrium. Second, the underlying stochastic difference equation (27) follows one particular lineage, lineage j, and the associated equilibrium distribution summarizes the possible states into which the members of lineage j can be born. Recall, however, that mortality is identically and independently distributed across both time and individuals, and that all individuals born in $t = 0$ are alike. Therefore, (28) must hold for all lineages, and the system exhibits asymptotic stationarity: with a continuum of individuals at any point in time and a long enough time horizon, the distibution of lifetime income of members of lineage j across time coincides with the distribution across all lineages at a given point in time.

Proposition 3 states the long-run levels of aggregate lifetime income, capital stock and consumption, respectively.

Proposition 3. ***Long-run Aggregates under Laissez-faire*** *Let $R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} < 1$. Then, aggregate lifetime income, capital stock and consumption in the laissez-faire economy converge to the long-run equilibrium values*

$$\begin{aligned} \omega(0) &= \lim_{t\to\infty}\int_0^1 (w+b_t^j)dj = \frac{w}{1-R\bar{a}\frac{q+\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}}} < \infty, && (29)\\ K(0) &= \lim_{t\to\infty} R\times\int_0^1 \bar{a}(w+b_t^j)dj = \bar{a}R\omega, && (30)\\ C(0) &= \lim_{t\to\infty}\int_0^1 c_t^j dj + \lim_{t\to\infty}(1-q)\int_0^1 c_{t+1}^j dj = \left((1-\bar{a}) + \frac{R\bar{a}(1-q)}{1+\gamma^{1/\sigma}}\right)\omega. && (31) \end{aligned}$$

Proof. Follows directly from the discussion in Vervaat (1979). (31) uses the fact that only the fraction $(1-q)$ of young workers survives to old age. The consumption-transfer allocation is derived in Appendix A. □

Figure B.2 displays the long-run distribution of the expected value of lifetime income for alternative values of R. Note that $\bar{a} > \bar{a}_e$ implies that the aggregates exceed their counterparts in the egoistic economy.[14]

4.2 Dynamics with the optimal scheme

Following Sheshinski and Weiss (1981), we rely on the assumption that each generation can freely decide on the contribution rate. Recalling (3), the individual's maximization problem is

$$\max_{a_t^j,\tau_t^j} U_t^j$$

[14] We can not obtain a similar result for the second moments. Applying the result in Section 5.2.2 of Vervaat (1979), for example the long-run capital distribution has variance $\frac{\bar{w}^2}{(1-E\bar{b}_t^2)(1-E\bar{b}_t)^2} \times \left(E\bar{b}_t^2 - (E\bar{b}_t)^2\right)$, which can exceed or falls short of its egoistic counterpart.

subject to

$$\begin{aligned}
c_t^j &= w(1-\tau_t^j) + b_t^j - a_t^j, && (32)\\
\left[c_{t+1}^{j,0}, b_{t+1}^{j,0}\right] &\equiv \arg\max u(c_{t+1}^j) + \nu(b_{t+1}^j) \text{ s.t. } c_{t+1}^j + b_{t+1}^j = a_t^j R + \frac{\tau_t^j w R}{1-q} && (33)\\
b_{t+1}^{j,1} &= a_t^j R && (34)\\
a_t^j &\geq 0,\ b_{t+1}^{j,0} \geq 0,\ c_{t+1}^{j,0},\ \tau_t^j \in [0,1]. && (35)
\end{aligned}$$

given b_t^j.

The first-order conditions with respect to a_t^j and τ_t^j may be written as

$$\begin{aligned}
u'(c_t^j) &\geq \beta(1-q)R\nu'(b_{t+1}^{j,0}) + \beta q \nu'(Ra_t^j)R \,(= \text{ if } a_t^j > 0), && (36)\\
u'(c_t^j) &= \beta R \nu'(b_{t+1}^{j,0}). && (37)
\end{aligned}$$

respectively. Combining (36) and (37) gives

$$\nu'(b_{t+1}^{j,0}) = \nu'(Ra_t^j), \tag{38}$$

so that no accidental bequests occur (i.e. the difference between inter-vivos transfers and the bequests in case of perematurе death is zero). A similar result is derived in Abel (1986) and Lockwood (2012). Fuster (2000) derives this condition for the bequest motive suggested by Barro (1974).[15] By virtue of the assumptions on u and ν, (37) implies $\tau^* \in (0,1)$. Moreover, the optimal contribution rate that brings about the desired equality of bequests across states in (38) is unique; for the LHS of (38) is monotonically decreasing in τ, while the RHS is independent of the contribution rate. Finally, by virtue of the old-age budget constraint, individuals use pension benefits exclusively to finance old age consumption, while the proceeds from conventional savings are used to finance intergenerational transfers.

Combining (36) and (38) yields

$$u'(c_t^j) = \beta R \nu'(Ra_t^j), \tag{39}$$

which determines the optimal fraction of overall savings to be privately saved. Since individuals are able to revise previous policies, the scheme affects intergenerational trade-offs. In particular, mortality (the only source of uncertainty in the model) vanishes in (39). The induced temporary allocation is, therefore, the result of the standard income and substitution effects stemming from an increased effective rate of return of the individual portfolio and an insurance effect, since the scheme allows equalization of bequest streams. Recalling the first-period budget constraint, the LHS of (39) is increasing in a_t^j. Since the RHS is decreasing in a_t^j, the lower Inada condition ensures uniqueness of the optimal decision $(a_t^j, \tau^{j,t})$.

Since all individuals at the beginning of time receive zero bequests, there will be intragenerational agreement on the contribution rate for all $t \geq 0$, so members of the

[15] In the pure altruistic case, one applies the envelope theorem to find the impact of the bequest on the maximum utility attainable by her heir born in $t+1$.

same generation receive the same amount of bequests in the presence of the scheme. In the presence of the scheme, therefore, there is no need to keep the index j.

Using the parametrization (22) and (23), the consumption-transfer ratio is independent of lifetime income, and the income expansion paths under full insurance for youth consumption, old-age consumption and both kind of intergenerational transfers are straight lines through the origin. From (38), $\tau_t w = a_t(1-q)/\gamma^{1/\sigma}$. Using this information in the first-period budget constraint and equation (39) gives optimal private savings as a function of initial wealth, $a_t = \bar{a}(\tau^*)(w + b_t)$, where $\bar{s}(\tau^*) = \bar{a}(\tau^*)(1 + (1-q)/\gamma^{1/\sigma})$; or, in closed form,

$$\bar{a}(\tau^*) = \frac{\gamma^{1/\sigma}}{1 - q + \gamma^{1/\sigma} + (\beta R^{1-\sigma})^{-1/\sigma}} \tag{40}$$

is the time-invariant fraction of lifetime income that is privately saved. The linearity of the model implies that the portfolio decision is independent of the overall savings decision. The sum of annuitized and non-annuitized savings is given by $s_t = \bar{s}(w + b_t^j)$, where

$$\bar{s}(\tau^*) = \left\{1 + \left[\beta R^{1-\sigma}(1 - q + \gamma^{1/\sigma})^{\sigma}\right]^{-1/\sigma}\right\}^{-1} \tag{41}$$

is the propensity to save out of lifetime income. Note that the fraction $\frac{1}{1+\gamma^{1/\sigma}/(1-q)}$ of the individual's portfolio is held as pension claims. We have therefore established

Lemma 5. ***Sequence of optimal contribution rates***

The sequence

$$\tau_t^* \;=\; \frac{\bar{a}_{\tau^*}(1-q)}{\gamma^{1/\sigma}} \frac{w + b_t}{w} = \bar{a}(\tau^*)(1-q)/\gamma^{1/\sigma} \times \sum_{i=0}^{t} \left(R\bar{a}(\tau^*)\right)^i . \tag{42}$$

converges from below to

$$\lim_{t\to\infty} \tau_t^* = \tau^* = \left[1 + \frac{(\beta R^{1-\sigma})^{-1/\sigma} + \gamma^{1/\sigma}(1-R)}{1-q}\right]^{-1} . \tag{43}$$

Proof. (42) is obvious from the above discussion. If $R\bar{a}(\tau^*) < 1$, then $\tau_t^* \to \frac{\bar{a}(\tau^*)(1-q)}{\gamma^{1/\sigma}(1-R\bar{a}(\tau^*))}$ as $t \to \infty$. Since the optimal split of the portfolio is independent of lifetime income, i.e. $\frac{\tau^* w}{\omega_t} = \bar{a}(\tau^*)(1-q)/\gamma^{1/\sigma})$, $\tau_{t+1} > \tau_t \, \forall t < \infty$. Finally, using (40) confirms (43). □

Note that Lemma 5 accounts for the transfer reduction induced by the scheme. The fact that we can define a sequence of non-zero optimal contribution rates should be contrasted with the seminal work of Barro (1974), who finds that government policy intended to change the intergenerational distribution of resources is neutral in welfare terms, so that the optimal contribution rate remains indeterminate. The present non-neutrality result is not driven by the conceptional difference in bequest motives discussed above. Rather, it is the existence of uninsured mortality risk that breaks the equivalence of Social Security pensions and private savings, a risk that does not appear in Barro (1974).

The essential result of the next corollary is that the sequence of optimal contribution rates in an unproductive economy populated by altruistic individuals converges to the time-consistent contribution rate of its egoistic counterpart.

Corollary 2. $\tau^* \gtreqless \tau_e^* = \frac{1}{1+\beta^{-1/\sigma}/(1-q)}$ *as* $R \gtreqless 1$.

The corollary states that τ^* is independent of the strength of the transfer motive when capital is unproductive. The economic intuition is as follows. Since the proceeds from annuitized savings are used for old-age consumption, the fraction of lifetime income saved in annuities is decreasing in the strength of the transfer motive. This tends to decrease the optimal contribution rate, given lifetime income. In turn, non-annuitized fraction of lifetime income in t increases with γ, and is transferred with interest to the next generation in $t+1$, independently of whether the individual survives to old age. Since contributions are wage-related, this effect tends to increase the optimal contribution rate. If capital is unproductive ($R = 1$), then both effects exactly cancel out in the long-run, so τ^* is independent of the strength of the bequest motive. In fact, a comparison of (16) and (43) establishes the results that long-run optimal contribution rate coincides with the one obtained in an economy populated with perfectly selfish individuals. Finally, since non-annuitized savings are transferred with interest, the latter (former) effect dominates the former (latter) as $R > 1$ ($R < 1$).

Since the scheme is fully funded, contributions to the scheme add to the economy-wide capital stock in the next period. Let $K(\tau^*)$ denote the long-run aggregate capital stock in the presence of the scheme. We have the following results:

Proposition 4. ***Convergence with the scheme***

If $R\bar{a}(\tau^*) < 1$, *then the economy with the scheme converges to the deterministic steady state with*

$$\begin{aligned} \omega(\tau^*) &= \lim_{t\to\infty}(w + b_t) = \frac{w}{1 - R\bar{a}(\tau^*)}, && (44)\\ K(\tau^*) &= \lim_{t\to\infty} K_t = \bar{s}(\tau^*)R\omega(\tau^*), && (45)\\ C(\tau^*) &= (1 - \bar{a}(\tau^*))\omega(\tau^*), && (46)\\ & && (47) \end{aligned}$$

Proof. By iteration, the sequence $K_{t+1} = R(a_t + \tau_t w) = [(1-q)/\gamma^{1/\sigma} + 1]\bar{a}(\tau)R\left(w + b_t^j\right) = \bar{s}w\sum_{i=0}^{t}(R\bar{a})^i + \bar{s}\,(\bar{a}R)^t$ converges if $R\bar{a}(\tau^*) < 1$. □

In the special case of unproductive capital, aggregate consumption is unaffected by the scheme and is equal to wage income w. If, moreover, $\beta^{-1} = R = \gamma = 1$, then the income expansion paths with respect to consumption and transfers are straight lines through the origin with slopes equal to one, implying a perfectly smooth consumption-transfer profile, $c_t(\tau^*) = c_{t+1}(\tau^*) = b_{t+1}(\tau^*) = a_t(\tau^*) = w/(2-q)$. Since $u = \nu$ in that case, the allocation sequence can be obtain ed without specifying the felicity function. In stationary equilirium, $K(\tau^*) = C(\tau^*) = w = C(0) < K(0)$, where $\tau^* = \frac{1-q}{2-q}\ \forall\sigma$. While the scheme contributes to capital accumulation, it unambiguously reduces the long-run capital stock. In general, however, the effect of the scheme on the long-run capital stock is ambigous *a priori*. To see why, the following lemma will be useful.

Lemma 6. ***(Propensities to save)***

1. *If $\sigma = 1$, then the propensity to save is unaffected by the scheme, $\bar{s}(\tau^*) = \bar{a}$.*
2. *If $\sigma < 1$, then the propensity to save in the economy with the scheme is higher than in the economy without it, $\bar{s}(\tau^*) > \bar{a}$.*

Proof. The first part follows directly from equations (26) and (41). Suppose $\sigma < 1$. Comparing (26) and (41) gives $\bar{s}(\tau) > \bar{a}$ if and only if

$$(1 - q + \gamma^{1/\sigma})^\sigma > (1 - q)(1 + \gamma^{1/\sigma})^\sigma + q\gamma.$$

Rewriting the LHS, we have $(1-q+\gamma^{1/\sigma})^\sigma = ((1-q)(1+\gamma^{1/\sigma})+q\gamma^{1/\sigma})^\sigma > ((1-q)(1+\gamma^{1/\sigma}))^\sigma$, $\forall\sigma > 0$. As to the RHS, $(1-q)(1+\gamma^{1/\sigma})^\sigma + q\gamma \leq \big((1-q)(1+\gamma^{1/\sigma})\big)^\sigma + q\gamma < \big((1-q)(1+\gamma^{1/\sigma})\big)^\sigma$; for $\sigma \leq 1$ by assumption. □

Intuitively, there are two components of the effective return to saving, namely, the expected return of the portfolio and the transfer insurance. Since the scheme is actuarially fair, the expected rate of return of the portfolio (including the premature death case) is simply R and therefore unaffected by the scheme, so the change in the propensity to save must be due to the bequest insurance effect. With bequest insurance, the marginal value of transferring one unit of consumption to the next period increases. In that sense, the bequest insurance appears to be an additional return to saving, and Lemma 6 states that if $\sigma < 1$ the effect of the transfer insurance on savings acts like a substitution effect stemming from an increase in the portfolio's return. The scheme increases the propensity to save out of lifetime income, which, in turn, tends to increase the long-run capital stock.

Moreover, while both annutized and non-annuitized savings add to the capital stock, the scheme reduces the fraction of initial wealth that is privately saved, i.e. $\bar{a}(\tau^*) < \bar{a}$, $t = 0, 1, 2, \ldots$. Only non-annuitized assets are transferred, and the resulting reduction in transfers received at birth tends to reduce the long run capital stock. Which of these opposing effects dominates is eventually a numerical question.[16]

[16]One might proceed instead by imposing $\sigma \to 1$, in which case the fraction of initial wealth saved for old age (in the form of both annuitized and non-annuitized assets) in the presence of the scheme equals the propensity to save under laissez faire ($\bar{s}(\tau^*) = \bar{a}(0)$). If, moreover, $R = 1$, then the scheme unambiguously decreases the long-run capital stock, i.e. $K(\tau) < K(0)$ for all $0 < \gamma \leq 1$.

5 Simulations: The cost of (in-)equality

The infinite series $w\left(1+\bar{b}_1+\bar{b}_1\bar{b}_2+\bar{b}_1\bar{b}_2\bar{b}_3+\ldots\right)$ converges quickly, particularly when the $\bar{b}_t$'s are small. It is thus fairly easy to generate approximate samples from the distribution of ω_∞ by taking a fairly short truncation. To be more precise, we follow one particular realization of the stochastic time series until $|\omega_t^j-\omega_{t+1}^j|<10^{-6}$. With $R=1$, for example, there is convergence within ten periods. A large sample of these stochastic elements is generated.

The parameters of the model economy are determined as follows. In order to maintain the two-period theoretical structure, let the economically active years start at the age of 20, and let one period span 40 years, implying that individuals reach retirement at the age of 60, the upper bound of life being 100 years. It seems tempting to impose $q=1/2$ as a benchmark as it implies a life expectancy at birth of 80 years. However, $q=1/2$ also implies that only one half, no less, of all 20 years-old fail to reach retirement. Since mortality rates begin to rise rapidly only after the age of 65 or 70, significantly lower values for q seem more suitable. We opt for the latter perspective and treat $q=0.3$ as a benchmark.

Let $\sigma=2$, which corresponds to the standard value of 1/2 for the intertemporal elasticity of substitution frequently found in the literature. By way of sensitivity analysis, Appendix C provides results for $\sigma=.5$. The remaining taste parameters are set so as to match the following targets: First, most of the literature on real business cycles employ a quarterly psychological discount factor of 0.9914 (see, e.g., Cooley and Prescott, 1995). Since premature mortality is an important determinant of subjective discounting in the present setup, let $\beta=0.35$ which, in conjunction with $q=0.3$, is equivalent to an effective subjective discount rate of $1/(\beta(1-q))-1$ per period, or 3.58% p.a.[17] Second, the strength of the transfer motive is set so as to match old age income from public transfers in per cent of their disposable income, $(\tau^*w/(1-q))/(a(\tau^*)+\tau^*w/(1-q))=1/(1+\gamma^{1/\sigma})$. Note that this number is independent of time preferences, mortality and the market rate of return. The rough average for OECD countries of about 2/3 serves as the reference, implying $\gamma=0.25$.

5.1 The long-run allocation

Table 1 reports the average allocations with and without the fully funded Social Security for alternative values of the probability premature death and the real rate of return on physical capital. One can think of the upper and lower panel, respectively, as separate islands, each with its particular q and R. Consider the constellation $(1-q,R)=(0.7,1)$ in the lower panel, which serves as the benchmark scenario.[18]

[17] β is usually chosen so as to match the market rate of return. In the current setup, however, the latter is exogenously given.

[18] Some data indicate an average annual real rate of return (reported as the lending interest rate adjusted for inflation) for most high income countries roughly between 2.5% and 7.5% p.a. in recent decades; see for example, the World Development Indicator data base, which covers the period 1961-

The remaining parameters yield Social Security expenditures of 29.3% of aggregate wage income, or 19.5% of gross domestic output, which are fairly close to the numbers in developed countries when one includes other pillars of Social Security such as health care and the like in the picture. The national savings rate in the long run equilibrium with the scheme is about 33%. In keeping with the theoretical results, aggregate consumption is unaffected by the scheme, but the national capital stock and wealth are reduced substantially: bequest wealth is 29% lower than under laissez-faire.[19] The magnitudes of both the wealth reduction and the ex ante insurance effect increase with R. For example, the coefficient of variation for lifetime income under laissez-faire increases from 0.136 with $R = 1$ to 0.432 with $R = 1.03^{40}$, in which case the equilibrium distribution has unbounded support.

5.2 Long-run welfare consequences

Table 1 also contains the long-run welfare consequences of the scheme. Following Kotlikoff and Spivak (1981) and Fehr and Habermann (2008), the latter are calculated as the percentage increase in initial wealth (which coincides with the expected present value of lifetime income in the current setup) that is necessary to make an individual living in the laissez-faire equilibrium as well off as in the equilibrium with the scheme. For individual $j \in [0, 1]$ born in t, this percentage solves $V_j^*(0, \phi^j \times \omega_t^j) = V^*(\tau^*, \omega_t(\tau^*))$, where ϕ^j is derived in the Appendix. Again, Rawls's (1973) veil of ignorance will be used to quantify the gains from insurance against the risk of being born into a particular initial wealth position. Therefore, I 'sum' over all individuals in the laissez-faire equilibrium to derive the long-run welfare consequence before the individual's type is revealed,[20]

$$\phi = \int_0^1 \phi^j dj. \tag{48}$$

In the linear model, the portfolio decision is independent of the decision on how much to carry over to old age (i.e., the choice of $s_t^j = a_t^j + \tau_t^j w$). Moreover, it can be seen in equations (40) and (42) that the individual's problem is scalable in wealth: Both the optimal purchase of pension claims as a fraction of total savings, $\left[1 + \gamma^{1/\sigma}/(1-q)\right]^{-1}$, and the propensity to save are independent of wealth. Since intergenerational transfers arise from previous savings, bequests received (and therefore the expected present value of lifetime income) by any individual j at the beginning of her first period of life are proportional to wage income, where the factor of proportionality is determined by the mortality history of her lineage. Since factor prices are fixed by assumption and unaffected by the scheme, this holds true with and without Social Security pensions, and the population average equivalent variation defined in (48) is invariant to shifts

2013. Since we are mainly interested in the scheme's very long-run welfare implications and the data span no more than 1.5 generations, relatively modest real interest rates seem appropriate.

[19]Recall that only the fraction $1 - q$ survives to old age, such that aggregate consumption in period t amounts to $c_t^* + (1-q)c_t^*$.

[20]Recall that an individual's family mortality history uniquely determines her wealth at birth if individuals are perfectly selfish. In that case, (48) 'sums' over all possible states.

Table 1: Laissez-faire and the optimal scheme: long-run (average) allocation and welfare effects, $\gamma = 0.25$, $\sigma = 2$

R	(1-q)	c_t^*	c_{t+1}^*	a_t^*	s_t^*	b_t^*	K/Y	b/Y	SSE	$\phi - 1$	$\phi_1 - 1$	$\phi_2 - 1$	$\phi_3 - 1$
$\tau = 0$													
1.00^{40}	0.5	0.819	0.361	0.542		0.361	0.351	0.234					
	0.6	0.776	0.368	0.553		0.328	0.356	0.211					
	0.7	0.735	0.372	0.559		0.294	0.358	0.188					
1.01^{40}	0.5	0.996	0.536	0.540		0.536	0.299	0.297					
	0.6	0.937	0.543	0.547		0.485	0.302	0.267					
	0.7	0.882	0.545	0.549		0.431	0.302	0.237					
1.02^{40}	0.5	1.267	0.830	0.564		0.832	0.251	0.370					
	0.6	1.179	0.832	0.565		0.744	0.251	0.331					
	0.7	1.094	0.824	0.559		0.653	0.250	0.292					
1.03^{40}	0.5	1.761	1.402	0.645		1.405	0.208	0.453					
	0.6	1.599	1.371	0.631		1.229	0.206	0.402					
	0.7	1.445	1.322	0.608		1.053	0.204	0.353					
$\tau = \tau^*$													
1.00^{40}	0.5	0.772	0.457	0.228	0.457	0.228	0.313	0.157	15.7	0.26	9.00	1.94	-10.64
	0.6	0.738	0.437	0.218	0.480	0.218	0.324	0.147	17.7	0.55	7.61	1.86	-8.92
	0.7	0.707	0.418	0.209	0.502	0.209	0.334	0.139	19.5	0.64	5.93	1.64	-6.93
1.01^{40}	0.5	0.890	0.642	0.216	0.431	0.321	0.263	0.196	13.1	-3.92	7.86	3.56	-15.34
	0.6	0.853	0.616	0.207	0.455	0.308	0.271	0.184	14.8	-2.84	6.70	3.38	-12.91
	0.7	0.819	0.591	0.199	0.477	0.296	0.279	0.173	16.3	-1.90	5.25	2.94	-10.09
1.02^{40}	0.5	1.043	0.917	0.208	0.415	0.459	0.217	0.239	10.8	-9.20	6.84	6.69	-22.75
	0.6	1.002	0.881	0.199	0.439	0.440	0.223	0.224	12.2	-7.14	5.85	6.22	-19.21
	0.7	0.963	0.847	0.192	0.460	0.423	0.228	0.210	13.3	-5.14	4.61	5.33	-15.08
1.03^{40}	0.5	1.201	1.347	0.207	0.413	0.674	0.176	0.287	8.8	-16.50	5.91	13.32	-35.73
	0.6	1.211	1.294	0.198	0.436	0.647	0.180	0.267	9.8	$^{\#}$-13.04	5.08	12.04	-30.16
	0.7	1.164	1.244	0.191	0.458	0.622	0.184	0.250	10.7	$^{\#}$-9.59	4.01	10.03	-23.63

The equivalent variation is reported as a % of the individual's expected present value of lifetime income.
SSE: Social Security expenditure (as % of gross domestic output).
$^{\#}$: unbounded support of the long-run laissez-faire equilibrium.

in the equilibrium wealth distribution. The strength of the bequest motive used to generate Table 1, $\gamma = 0.25$, suggests an ex ante welfare gain of 0.64%. To put this number in perspective, the remainder discusses the relative importance of insurance and the strength of the transfer motive.

5.2.1 Disentangling insurance

In the current setup, the scheme provides only transfer insurance and the induced ex ante insurance manifested in a more concentrated long-run wealth distribution.[21] To quantify the relative importance of both forms of insurance, the overall welfare effect is split into effects stemming from the reduction in wealth, transfer insurance, and ex ante insurance of the state at birth, respectively. First, the average willingness to pay for transfer insurance is calculated as follows. Suppose that each individual j in the laissez-faire equilibrium enjoys unchanged initial wealth, but also bequest insurance through the possibility of annuitization by means of the scheme. Denote the associated welfare number by $\phi_1 = \int \phi_1^j dj$. Since the inital wealth distribution is unaltered by the scheme, the transfer insurance effect coincides with the short-run welfare effect from the scheme. While the smoothing of transfers across states is beneficial to risk-averse individuals, an increase in the probability of surviving into old age reduces the gains from transfer insurance. At the extreme, $q = 0$ implies that the contribution to the scheme is a perfect substitute for private savings in physical capital (as in Barro (1974)) and such gains vanish. Second, consider the ex ante risk of being born into a particular state. The average percentage increase in initial wealth necessary to make the individuals in the laissez faire equilibrium as well off as they would be if they were faced with the ex ante expected initial wealth position, ϕ_2^j, solves $V(\phi_2^j(w + b_t^j)) = V(\int (w + b_t^j)dj)$. If transfers are motivated by the joy of giving, then individuals do not (fully) take into account the scheme's impact on future generations well-being, thereby also generating an externality in the form of wealth reduction. This effect is obtained as the residual $\phi_3 = \phi - \phi_1 - \phi_2$.

The overall welfare gain is decreasing in the market rate of return. For modest real rate of returns, transfer insurance considerably dominates ex ante insurance. However, since bequests are transferred with interest, an increase in R increases the spread in the long-run wealth distribution, so that ex ante insurance becomes increasingly important. Consider, again, the benchmark constellation $(1 - q, R) = (0.7, 1.03^{40})$, in which case the support of the laissez-faire equilibrium distribution is unbounded, and the contribution of ex ante insurance to the overall welfare effect is 2.5 times the contribution of transfer insurance. However, the increase in R also enhances the reduction in capital and wealth, which is the far most important contributor to the overall effect if R is high.

By virtue of Lemma 6 the overall propensity to save with the scheme is higher than without it whenever $\sigma < 1$. To illustrate what happens in this range, Table C.2 repli-

[21] The model setup is unambiguous at this point: the two period assumption implies that the annuity is only paid once, excluding the insurance of longevity risk (i.e. the risk of running out of resources). Recalling footnote 6, insurance of lifetime risk itself is also excluded.

cates the allocative and welfare consequences of the scheme for 0.5, which corresponds to a relatively high intertemporal elasticity of substitution of 2. The increase in long-run capital induced by the scheme now goes along with a long-run reduction in welfare; for the ex ante insurance is significantly reduced (for the unproductive panel, the ratio of ex ante insurance bequest insurance gains is 0.062, compared to 0.28 in the case where $\sigma = 2$).

5.2.2 Welfare gains and the strength of the bequest motive

Presuming $R = 1$, Figure B.3 illustrates that the welfare consequences of the scheme depend qualitatively on the strength of the bequest motive. The red line is the net welfare effect as a function of the strength of the bequest motive. While altruists gain far less than egoistists from the scheme, the crowding out induced by the scheme is compatible with long-run welfare gains as long as the strength of the transfer motive is low, although the reduction in transfer wealth is quite substantial. However, overall welfare gains turn into losses if the transfer motive is strong enough: while egoists (i.e. $\gamma = 0$) gain on average 3.29% of their expected lifetime income, individuals who value inter vivos transfers and old-age consumption alike (i.e. $\gamma = 1$) suffer from a modest average welfare loss of about -0.1% of expected lifetime income. The driving forces are transfer insurance (in the egoistic economy, a higher effective rate of return) and the crowding out of wealth. Both effects are decreasing in γ. Since $R = 1$, the laissez-faire equilibrium distribution is bounded (see Proposition 3), such that ex ante insurance is relatively insensitive to the strength of of the bequest motive. While ex ante insurance plays a minor role, it may tip the scales in qualitative evaluations of overall welfare consequences from Social Security pensions.

5.2.3 Long-run welfare consequences: the egoistic economy

Welfare effects crucially depend on whether the transfer motive is operative. Table C.1 provides the welfare numbers for the egoistic economy. Individuals save purely out of perfect selfishness. However, the scheme entails an increase in the effective rate of return, inducing standard income and substitution effects. The latter are bundled in the number ϕ_1 (equivalently, in the first bar for egoistic individuals in Figure B.3). Not surprisingly, ϕ_1 is decreasing in the probability of surviving into old age; for the effective rate of return is decreasing in $(1 - q)$. If capital is unproductive, then, on average, the income effect stemming from the higher return is of the same size in absolute value as the income effect stemming from the elimination of bequests. In the benchmark $1 - q = 0.7$, the remaining substitution effect falls short of the ex ante insurance effect: $11.76 - 10.53 = 1.23 < 2.06$. As noted above, egoists gain much more from the scheme than their altruistic counterparts. At first glance, one might argue that the optimal scheme and the associated welfare gains are smaller in size; for the bequest motive reduces the optimal degree of annuitization. By virtue of Corollary 2, however, the long-run optimal contribution rate in the altruistic economy coincides with the one obtained in the egoistic economy if capital is unproductive. Given the insensitiveness of ex ante insurance to γ, the increase in the effective rate of return is much more valued by egoists than bequests insurance is valued by altruists.

6 Conclusions

The paper analyzed the consequences from insuring mortality risk by means of standard fully funded Social Security Pensions in two socio-economic setups: an egoistic economy in which individuals save out of a pure life-cycle motive, and an altruistic economy in which individuals save out of both a life-cycle motive and an altruistic motive which reflects the joy of giving. In the former setup, the pension system promises a higher effective return than the market. In the latter setup, the pension system is an instrument to smooth intergenerational transfers across the states death and survival. Whatever the individuals' attitudes towards their heirs, the pension system reduces intergenerational transfers, resulting in a lower but more concentrated distribution of lifetime income in the long-run.

Altruistic behaviour is commonly seen to reduce the fraction of lifetime income to be optimally annuitized. Given an equality of the market interest rate and the population growth rate, our model implies that, once the reduction in intergenerational transfer is taken into account, the long-run optimal contribution rate is independent of the strength of the transfer motive, and coincides with the one associated with the egoistic economy. If capital is productive, then the pension system's long-run optimal contribution rate is even higher if individuals are altruistic, reflecting the desire to make transfers to the heir.

The short-run welfare effects are clear-cut. Egoists gain from the higher effective rate of return, while altruists gain from transfer insurance. The long-run welfare consequences are not that obvious a priori. While the pension system reduces intergenerational transfers, it also reduces the mortality-related ex ante risk of being born with a particular amount of wealth. The model suggests that, once ex ante insurance is taken into account, fully funded Social Security Pensions can not be rejected a priori: if capital is not too productive and the bequest motive is not too strong, then the scheme generates long-run welfare gains; for insurance gains outweigh the crowding out of within-family transfers.

A Equivalent variation

The consumption-transfer allocation under laissez-faire, namely,

$$\begin{aligned} c_t^{j,0} &= (1-\bar{a})(w+b_t^j) && \text{(A.1)}\\ b_{t+1}^{j,0} &= \frac{\gamma^{1/\sigma}}{1+\gamma^{1/\sigma}} R\bar{a}(w+b_t^j) && \text{(A.2)}\\ b_{t+1}^{j,1} &= R\bar{a}(w+b_t^j) && \text{(A.3)}\\ c_{t+1}^{0} &= \frac{1}{1+\gamma^{1/\sigma}} R\bar{a}(w+b_t) && \text{(A.4)} \end{aligned}$$

gives the indirect utility of individual j born in period t as

$$V_t^j(0) = \bar{v}(w+b_t^j)^{1-\sigma}, \tag{A.5}$$

where

$$\bar{v} = \left[(1-\bar{a})^{1-\sigma} + \beta\left(\bar{a}R\right)^{1-\sigma}(1-q)\left((1+\gamma^{1/\sigma})^{\sigma} + q/(1-q)\gamma\right)\right]/(1-\sigma), \tag{A.6}$$

which approaches $(1-\bar{a}_e)^{1-\sigma}\left(1+\left(\beta(1-q)R^{1-\sigma}\right)^{1/\sigma}\right)/(1-\sigma)$ as $\gamma\to 0$.

With the scheme, the consumption-transfer allocation

$$\begin{aligned} c_t^{0} &= (1-\bar{s}(\tau^*))(w+b_t) && \text{(A.7)}\\ b_{t+1}^{0} = b_{t+1}^{s} &= R\bar{a}(\tau^*)(w+b_t) = a_t R && \text{(A.8)}\\ c_{t+1}^{0} &= \gamma^{-1/\sigma} R\bar{a}(\tau^*)(w+b_t) && \text{(A.9)} \end{aligned}$$

gives the indirect utility of a member of the generation born in t as

$$V_t(\tau^*) = \bar{v}(\tau^*)(w+b_t)^{1-\sigma}, \tag{A.10}$$

where

$$\bar{v}(\tau^*) = \left[(1-\bar{s}(\tau^*))^{1-\sigma} + \beta(1-q)\left(\bar{a}(\tau^*)R/\gamma^{1/\sigma}\right)^{1-\sigma} + \beta\gamma\left(\bar{a}(\tau^*)R\right)^{1-\sigma}\right]/(1-\sigma), \tag{A.11}$$

which approaches $(1-\tau_e^*)^{1-\sigma}\left(1+\left(\beta R^{1-\sigma}\right)^{1/\sigma}(1-q)\right)/(1-\sigma)$ as $\gamma\to 0$.

Recalling the definition of ϕ^j, the equivalent variation reads

$$\phi_t^j = \left(\frac{\bar{v}(\tau^*)}{\bar{v}}\right)^{\frac{1}{1-\sigma}} \omega(\tau^*)/\omega_t^j \frac{w+b_t^{\tau}}{w+b_t^j}. \tag{A.12}$$

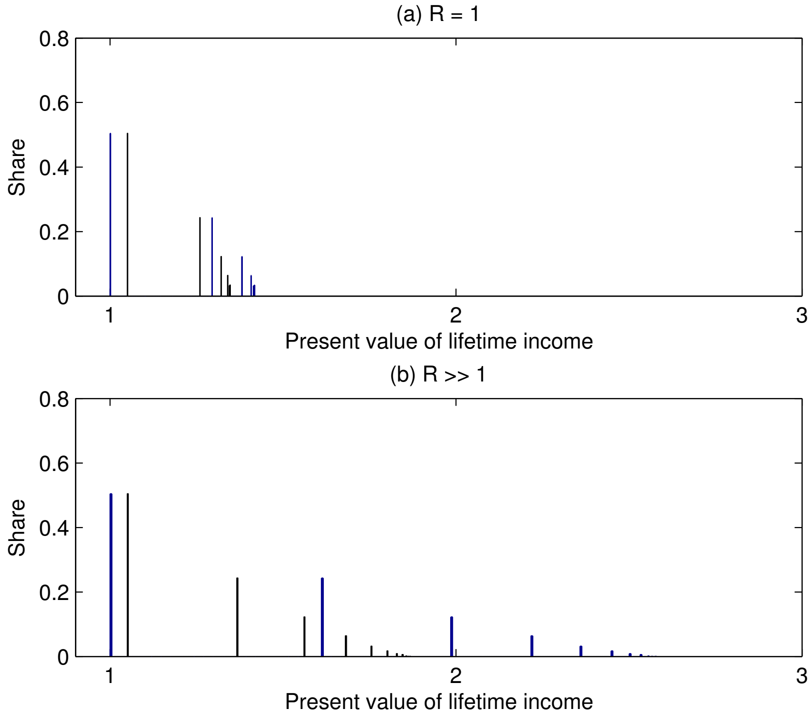
For the egoistic economy, the equivalent variation of an individual whose j previous, consecutive forebears within the lineage died prematurely is given by

$$\phi^j = \left[\frac{1+\left(\beta(1-q)R^{1-\sigma}\right)^{1/\sigma}}{1+(1-q)\left(\beta R^{1-\sigma}\right)^{1/\sigma}}\right] \times \left[\frac{1+\left(\beta R\right)^{1/\sigma}}{1+(1-q)\left(\beta R\right)^{1/\sigma}}\right]^{\frac{1}{1-\sigma}} \times \frac{w}{w+b^j}, \tag{A.13}$$

where $w+b^j = w\sum_{i=0}^{j}\left(\bar{a}R\right)^i$. Recalling that there are $q^j(1-q)$ type-j individuals in long-run equilibrium, the aggregate welfare effect stemming from the scheme is $\phi = \sum_{j=0}^{\infty} q^j(1-q)\phi^j$, where the double sum converges, provided that the long-run equilibrium exists, i.e. $\bar{a}Rq < 1$.

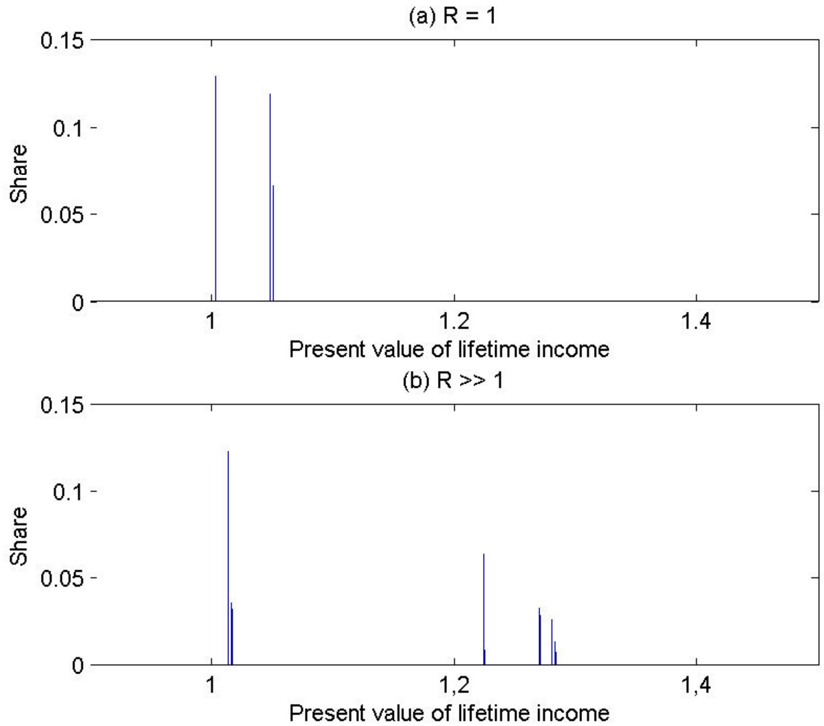
B Plots

Figure B.1: Long-run distributions of lifetime income: the egostic economy



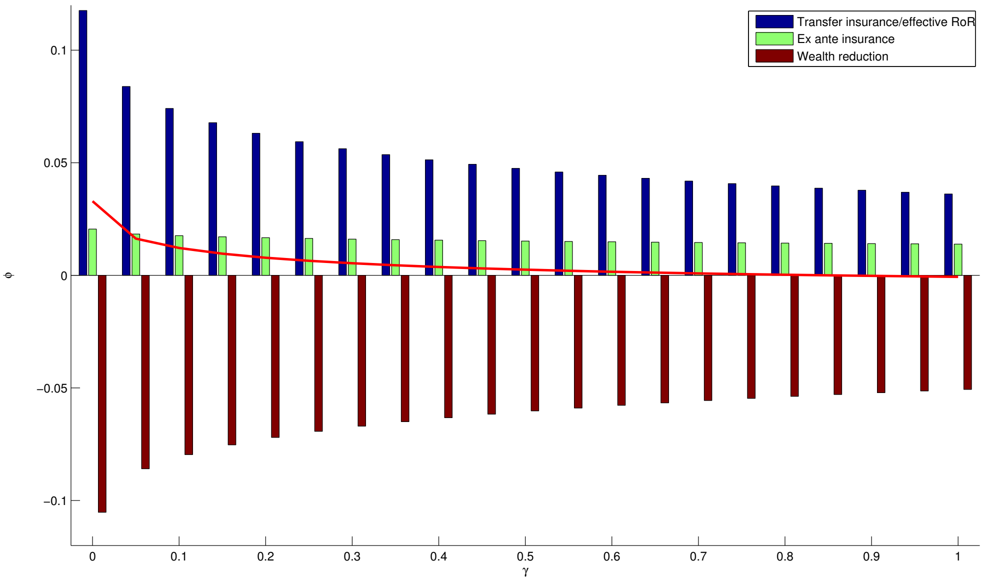
The figure illustrates the distributions of lifetime income in the stationary equilibrium with (black bars) and without the scheme (blue bars). The algorithm used to produce the figure is discussed in Section 5. The parameters are $q = 0.5$, $\beta = 0.5$, $\sigma = 2$, $R = 1.02^{40}$, $w = 1$.

Figure B.2: Long-run distributions of lifetime income: the altruistic economy



The figure illustrates the distributions of lifetime income in the stationary equilibrium of the altruistic economy in the absence of the scheme. The algorithm used to produce the figure is discussed in Section 5. The parameters are $q = 0.5$, $\beta = 0.5$, $\sigma = 2$, $\gamma = 0.25$, $R = 1.02^{40}$, $w = 1$.

Figure B.3: Population average equivalent variation as a function of the strength of the Transfer motive



The figure illustrates the welfare consequences of the optimal scheme for different values of γ when capital is unproductive, i.e. $R = 1$. The red line is the net effect. The bars represent a decomposition of the equivalent variation into bequest insurance, ex ante insurance and crowding out of wealth. The first bundle of bars (on the far left) represents the decomposition of the equivalent variation for the egoistic economy (i.e. $\nu(\cdot) = 0$) into (i) income and substitution effects induced by an increase in the effective rate of return, (ii) ex ante insurance and (iii) capital reduction. The parameters use are $R = 1$, $q = 0.3$, $\beta = 0.35$, $\sigma = 2$.

C Tables

Table C.1: Long-Run Welfare Effects in the Egoistic Economy

σ	R	(1-q)	$\phi - 1$	$\phi_1 - 1$	$\phi_2 - 1$	$\phi_3 - 1$
2	1.00^{40}	0.5	4.58	19.81	2.30	-17.53
		0.6	4.04	15.83	2.30	-14.09
		0.7	3.29	11.76	2.06	-10.53
	1.01^{40}	0.5	-1.57	16.82	3.94	-22.33
		0.6	-0.68	13.55	3.89	-18.11
		0.7	0.09	10.14	3.44	-13.66
	1.02^{40}	0.5	-7.60	14.23	6.72	-28.55
		0.6	-5.30	11.54	6.49	-23.32
		0.7	-3.40	8.69	5.64	-17.72
	1.03^{40}	0.5	-13.44	12.02	11.46	-36.91
		0.6	-9.74	9.79	10.77	-30.30
		0.7	-6.57	7.42	9.14	-23.12
0.5	1.00^{40}	0.5	1.48	2.97	0.02	-1.52
		0.6	1.17	2.82	0.04	-1.68
		0.7	0.85	2.43	0.06	-1.64
	1.01^{40}	0.5	1.11	4.36	0.11	-3.36
		0.6	0.60	4.11	0.20	-3.70
		0.7	0.23	3.52	0.30	-3.58
	1.02^{40}	0.5	-0.56	6.33	0.50	-7.39
		0.6	-1.26	5.92	0.89	-8.07
		0.7	-1.47	5.02	1.27	-7.75
	1.03^{40}	0.5	-4.83	9.08	2.32	-16.24
		0.6	-5.29	8.38	3.96	-17.64
		0.7	-4.62	7.02	5.29	-16.92

Table C.2: Laissez-faire and the optimal scheme: long-run (average) allocation and welfare effects (in %), $\gamma = 0.25$, $\sigma = 0.5$

R	(1-q)	c_t^*	c_{t+1}^*	a_t^*	s_t^*	b_{t+1}^*	K/Y	b/Y	SSE	$\phi - 1$	$\phi_1 - 1$	$\phi_2 - 1$	$\phi_3 - 1$
$\tau = 0$													
1.00^{40}	0.5	0.977	0.046	0.049		0.026	0.047	0.025					
	0.6	0.965	0.128	0.061		0.026	0.058	0.024					
	0.7	0.950	0.153	0.074		0.024	0.069	0.022					
1.01^{40}	0.5	0.984	0.103	0.074		0.058	0.066	0.052					
	0.6	0.966	0.128	0.091		0.057	0.080	0.051					
	0.7	0.944	0.153	0.109		0.053	0.094	0.046					
1.02^{40}	0.5	1.018	0.235	0.113		0.131	0.090	0.105					
	0.6	0.991	0.288	0.138		0.130	0.106	0.099					
	0.7	0.955	0.341	0.164		0.118	0.120	0.087					
1.03^{40}	0.5	1.134	0.571	0.186		0.320	0.116	0.200					
	0.6	1.087	0.689	0.224		0.312	0.130	0.180					
	0.7	1.021	0.795	0.259		0.280	0.141	0.152					
$\tau = \tau^*$													
1.00^{40}	0.5	0.942	0.115	0.007	0.065	0.007	0.061	0.007	5.42	-0.00	1.78	0.05	-1.84
	0.6	0.932	0.114	0.007	0.076	0.007	0.070	0.007	6.37	-0.10	1.69	0.07	-1.86
	0.7	0.921	0.113	0.007	0.086	0.007	0.079	0.007	7.27	-0.13	1.46	0.09	-1.68
1.01^{40}	0.5	0.921	0.250	0.011	0.095	0.016	0.083	0.014	7.36	-1.25	2.59	0.24	-4.08
	0.6	0.906	0.246	0.010	0.110	0.015	0.094	0.013	8.52	-1.30	2.44	0.34	-4.07
	0.7	0.891	0.242	0.010	0.124	0.015	0.105	0.013	9.61	-1.17	2.09	0.41	-3.68
1.02^{40}	0.5	0.897	0.536	0.015	0.137	0.034	0.105	0.026	9.32	-4.21	3.71	1.11	-9.03
	0.6	0.876	0.523	0.015	0.157	0.033	0.117	0.024	10.56	-3.98	3.47	1.51	-8.96
	0.7	0.856	0.511	0.014	0.176	0.032	0.127	0.023	11.67	-3.34	2.96	1.75	-8.05
1.03^{40}	0.5	0.875	1.140	0.022	0.197	0.071	0.120	0.043	10.65	-10.14	5.23	5.25	-20.63
	0.6	0.845	1.102	0.021	0.224	0.069	0.129	0.040	11.71	-8.88	4.85	6.66	-20.38
	0.7	0.818	1.066	0.020	0.269	0.067	0.137	0.037	12.62	$^{\#}$-6.99	4.08	7.21	-18.28

The equivalent variation is reported as a % of the individual's expected present value of lifetime income.
SSE: Social Security expenditure (as % of gross domestic output).
$^{\#}$: unbounded support of the long-run laissez-faire equilibrium.

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