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Abstract

This paper employs the unrestricted extended constant conditional correlation GARCH specification proposed in Conrad and Karanasos (2008) to examine the intertemporal relationship between the uncertainties of inflation and output growth in the US. We find that inflation uncertainty effects output variability positively, while output variability has a negative effect on inflation uncertainty.

Keywords: Bivariate GARCH process, negative volatility feedback, inflation uncertainty, output variability.

JEL Classification: C32, C51, E31.

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1 Introduction

During the last decade researchers have employed various bivariate GARCH-in-mean models to investigate the relationship between the uncertainties of inflation and output growth and/or to examine their impact on the levels of inflation and growth (see, for example, Grier et al., 2004).[1]

The two most commonly used specifications are the diagonal constant conditional correlation (DCCC) model (see, for example Grier and Perry, 2000, Fountas et al., 2006, and Fountas and Karanasos, 2007) and the BEKK representation (see, for example, Lee, 1999, and Grier and Grier, 2006). However, these two specifications are characterized by rather restrictive assumptions regarding potential volatility spillovers. At the one extreme, the former assumes that there is no link between the two uncertainties, whereas, near the other extreme, the latter only allows for a positive variance relationship.

In sharp contrast, several economic theories predict either a *positive* or a *negative* association between the variabilities of inflation and growth (for more details and a review of the literature, see, Karanasos and Kim, 2005). Obviously, the extent to which there is an interaction of either sign between the two variances is an issue that cannot be resolved on merely theoretical grounds. These considerations reinforce a widespread awareness of the need for more empirical evidence, but also make clear that a good empirical framework is lacking.

In this paper we employ the unrestricted extended CCC (UECCC) GARCH model to examine how the US nominal and real uncertainties are interrelated. This specification, defined in Conrad and Karanasos (2008), allows for feedback effects between the two volatilities that can be of either sign, i.e. positive or negative.[2] More specifically, Conrad and Karanasos (2008) derive necessary and sufficient conditions which ensure the positive definiteness of the conditional covariance matrix even in the case of negative volatility feedback. While negative values of the GARCH coefficients have commonly been thought of as resulting either from sampling error or model misspecification, they show that this is not necessarily the case. Interestingly, negative volatility spillovers may be in line with economic theory.

This is the first paper to apply this flexible bivariate formulation to investigating the relation between the variabilities of US inflation and output. We find strong evidence supporting the Logue and Sweeney (1981) theory that inflation uncertainty has a positive impact on the volatility of growth. In sharp contrast, real variability affects nominal uncertainty negatively as predicted by, among others, Fuhrer (1997). Clearly, this negative effect could not have been detected by applying the restrictive DCCC or BEKK GARCH specifications.

[1] We will use the terms variance, variability, uncertainty and volatility interchangeably in the remainder of the text.

[2] The specification is termed 'unrestricted extended' because it can be viewed as an unrestricted version of the extended CCC (ECCC) specification of Jeantheau (1998) which allows for positive volatility feedback only.

2 The Bivariate GARCH Model

We use a bivariate model to simultaneously estimate the conditional means, variances, and covariances of inflation and output growth. Let $\mathbf{Y}_t = (\pi_t \ y_t)'$ represent the 2×1 vector with the inflation rate and real output growth. The symbols $\odot$ and $^\wedge$ denote the Hadamard product and the elementwise exponentiation respectively. Further, $\mathcal{F}_{t-1} = \sigma(\mathbf{Y}_{t-1}, \mathbf{Y}_{t-2}, \ldots)$ is the filtration generated by the information available up through time $t-1$.

Define the residual vector as $\boldsymbol{\varepsilon}_t = (\varepsilon_{\pi,t} \ \varepsilon_{y,t})' = \mathbf{z}_t \odot \mathbf{h}_t^{\wedge 1/2}$, where the stochastic vector $\mathbf{z}_t = (z_{\pi,t} \ z_{y,t})'$ is independent and identically distributed (*i.i.d.*) with mean zero, finite second moments, and 2×2 correlation matrix $\mathbf{R} = [\boldsymbol{\rho}_{ij}]_{i,j=\pi,y}$ with diagonal elements equal to one and off-diagonal elements absolutely less than one. $\mathbf{h}_t = (h_{\pi,t} \ h_{y,t})'$ denotes a vector of $\mathcal{F}_{t-1}$ measurable conditional variances.

We estimate the following bivariate AR(p)-GARCH$(1,1)$-in-mean model

$$\mathbf{Y}_t = \boldsymbol{\Gamma}_0 + \sum_{l=1}^{p} \boldsymbol{\Gamma}_l \mathbf{Y}_{t-l} + \sum_{r=0}^{s} \boldsymbol{\Delta}_r \mathbf{h}_{t-r} + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{I}$ is the 2×2 identity matrix, $\boldsymbol{\Gamma}_0 = [\gamma_i]_{i=\pi,y}$, $\boldsymbol{\Gamma}_l = [\gamma_{ij}^{(l)}]_{i,j=\pi,y}$ and $\boldsymbol{\Delta}_r = [\delta_{ij}^{(r)}]_{i,j=\pi,y}$. We assume that the roots of $\left|\mathbf{I} - \sum_{l=1}^{p} \boldsymbol{\Gamma}_l L^l\right|$ lie outside the unit circle. Note that our specification allows the conditional variances to effect the level variables with some time delay r.

Following Conrad and Karanasos (2008), we impose the UECCC GARCH(1,1) structure on the conditional variances:

$$\mathbf{h}_t = \boldsymbol{\omega} + \mathbf{A}\boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \mathbf{B}\mathbf{h}_{t-1}, \tag{2}$$

where $\boldsymbol{\omega} = [\omega_i]_{i=\pi,y}$, $\mathbf{A} = [a_{ij}]_{i,j=\pi,y}$ and $\mathbf{B} = [b_{ij}]_{i,j=\pi,y}$.[3]

Finally, we assume that the above model is minimal in the sense of Jeantheau (1998, Definition 3.3) and invertible (see Assumption 2 in Conrad and Karanasos, 2008). The invertibility condition implies that the inverse roots of $|\mathbf{I} - \mathbf{B}L|$, denoted by ϕ_1 and ϕ_2, lie inside the unit circle.

Conrad and Karanasos (2008) show that the following four conditions are necessary and sufficient for $\mathbf{h}_t > 0$ for all t: (i) $(1 - b_{yy})\omega_\pi + b_{\pi y}\omega_y > 0$ and $(1 - b_{\pi\pi})\omega_y + b_{y\pi}\omega_\pi > 0$, (ii) ϕ_1 is real and $\phi_1 > |\phi_2|$, (iii) $\mathbf{A} \geq 0$ and (iv) $[\mathbf{B} - \max(\phi_2, 0)\mathbf{I}]\mathbf{A} > 0$. These constraints do not place any *a priori* restrictions on the signs of the coefficients in the $\mathbf{B}$ matrix. In particular, this implies that potential negative volatility spillovers are allowed.[4]

3 Economic Theory

Various economic theories make predictions about the link between the volatilities of inflation and output on the one hand and the effects of these uncertainties on the levels of the

[3] This specification nests the DCCC model when $\mathbf{A}$ and $\mathbf{B}$ are diagonal matrices and Jeantheau's (1998) ECCC model when $a_{ij} \geq 0$ and $b_{ij} \geq 0$.

[4] Nakatani and Teräsvirta (2008) derive a similar set of conditions for the more restricted case that the two diagonal elements of the GARCH matrix are positive.

Table 1: Economic Hypothesis.

Panel A: volatility feedback	+	−
$b_{\pi y}$	Devereux (1989)	Fuhrer (1997)
$b_{y\pi}$	Logue-Sweeney (1981)	Fuhrer (1997)
Panel B: in-mean effects	+	−
$\delta^{(r)}_{\pi\pi}$	Cukierman and Meltzer (1986)	Holland (1995)
$\delta^{(r)}_{\pi y}$	Devereux (1989)	Taylor (1979)
$\delta^{(r)}_{y\pi}$	Dotsey and Sarte (2000)	Friedman (1977)
$\delta^{(r)}_{yy}$	Black (1987)	Pindyck (1991)

Note: Standard business cycle models imply that there is no link between the level of output and output variability and, hence, imply $\delta^{(r)}_{y\pi} = 0$.

respective variables on the other hand. For example, according to Fuhrer (1997) there exists a trade-off between nominal uncertainty and real variability. However, in Devereux's (1989) model higher output variability is associated with more inflation uncertainty. Similarly, models with a stable inflation-unemployment trade-off imply a positive relationship between nominal and real uncertainty (see, Logue and Sweeney, 1981). We refrain from discussing the various economic theories in detail and instead present in Table 1, Panel A, a summary of the signs implied by the respective theories about volatility interactions for the parameters of our empirical model. Table 1, Panel B, includes the economic theories regarding the effects of the uncertainties on the level variables. A detailed survey of the theories is provided, e.g., in Karanasos and Kim (2005) and Fountas and Karanasos (2007).

4 Empirical Results

Monthly data, obtained from Datastream, are used to provide a reasonable number of observations. The inflation and output growth series are calculated as the monthly difference in the natural log of the Consumer Price Index and Industrial Production Index, respectively. The data range from 1960:01 to 2007:12 and, hence, comprise 576 usable observations. Applying various unit root tests to both series, we came to the conclusion that inflation as well as output growth can be treated as stationary variables.

Within the bivariate UECCC GARCH-in-mean framework we will analyze the dynamic adjustments of the conditional variances of US inflation and output growth, as well as the implications of these dynamics for the direction of causality between the two uncertainties. Parameter estimates were obtained by quasi maximum likelihood estimation (QMLE). To check for the robustness of our estimates we used a range of starting values and, hence, ensured that the estimation procedure converged to a global maximum. The best model was chosen on the basis of Likelihood Ratio (LR) tests and three alternative information criteria. For reasons of brevity, we refrain from presenting the estimation results for the autoregressive parameters; instead, in Table 2 we concentrate on the main parameters of interest.

As we are most interested in the potential spillover effects between the two volatilities, we first discuss the implications of equation (5) in Table 2.[5] First, since $b_{y\pi}$ is positive and significant there is strong evidence that nominal uncertainty has a positive impact on real volatility, as predicted by Logue and Sweeney (1981). Second, the negative and significant value of $b_{\pi y}$ indicates that output variability affects the uncertainty of inflation negatively and provides support for the Fuhrer (1997) theory.[6] Most importantly, although one off-diagonal GARCH coefficient is negative the necessary and sufficient conditions for the positive definiteness of the conditional covariance matrix are satisfied.

Table 2: Parameter Estimates for the AR-UECCC-GARCH$(1,1)$-in-mean model.

$$\pi_t = \ldots + \underset{(0.043)}{0.130} h_{\pi,t-1} - \underset{(0.004)}{0.006} h_{y,t} + \varepsilon_{\pi,t} \tag{3}$$

$$y_t = \ldots - \underset{(0.081)}{0.137} h_{\pi,t-1} + \underset{(0.006)}{0.017} h_{y,t-3} + \varepsilon_{y,t} \tag{4}$$

$$\mathbf{h}_t = \begin{pmatrix} \underset{(0.119)}{0.3816} \\ \underset{(3.098)}{5.3901} \end{pmatrix} + \begin{pmatrix} \underset{(0.013)}{0.0684} & \underset{(0.003)}{0.0064} \\ - & \underset{(0.051)}{0.2945} \end{pmatrix} \boldsymbol{\varepsilon}_{t-1}^{\wedge 2} + \begin{pmatrix} \underset{(0.028)}{0.9145} & \underset{(0.004)}{-0.0077} \\ \underset{(0.345)}{1.0250} & \underset{(0.058)}{0.5259} \end{pmatrix} \mathbf{h}_{t-1}, \tag{5}$$

with $\rho = -0.0396$ (0.0449).

Notes: This table reports parameter estimates of the bivariate AR-UECCC-GARCH$(1,1)$-in-mean model for the US inflation (π_t) and output (y_t) data. $h_{\pi,t}$ and $h_{y,t}$ denote the conditional variances of inflation and output, respectively. The numbers in parenthesis are robust standard errors.

Finally, the parameter estimates in equation (3) in Table 2 show that higher nominal uncertainty leads to higher inflation rates as suggested by Cukierman and Meltzer (1986), while increasing output volatility appears to lower the average inflation rate (the so-called Taylor effect). The results in equation (4) support the Friedman (1977) hypothesis that increasing inflation uncertainty has a negative effect on output growth.[7] Moreover, higher real variability appears to increase output growth as predicted by Black (1987). Note that in three out of the four cases the effects from the uncertainties to the levels arise with some time delay (insignificant contemporaneous parameters are not presented), which is

[5] At first sight, the difference in the size of the estimated $b_{\pi y}$ and $b_{y\pi}$ coefficients is surprising. However, these differences can be explained by the fact that the variance in output is much higher than in inflation. In particular, the descriptive analysis of the two series shows that the variance of output is about five times higher than that of inflation.

[6] Both results are in line with the findings in Karanasos and Kim (2005) for the US. However, in contrast to our one-step estimation approach, Karanasos and Kim (2005) employed an inefficient two-step strategy.

[7] Both cross effects, i.e., the coefficients $\delta_{\pi y}^{(0)}$ and $\delta_{y\pi}^{(1)}$ are significant at the 10% level.

to be expected when working with monthly data.[8]

In order to check for the robustness of our results and to control for possible policy changes we split our sample into the sub-periods 1960-1979 and 1980-2007. While our conclusions regarding the link between the variabilities of inflation and output remained unchanged, it appears that the effects from nominal and real uncertainty on output growth are stronger in the second sub-period.[9]

5 Conclusions

This is the first paper which employs the UECCC GARCH model to investigate the inflation-growth uncertainty link using US data. The main advantage of this new specification is that it allows for volatility feedback of either sign, i.e., positive or negative. Thus, we are able to test economic theories which suggest a trade-off between the variabilities of inflation and output. Our results show that real volatility affects nominal uncertainty negatively, as predicted by Fuhrer (1997). In sharp contrast, we find strong evidence supporting the Logue and Sweeney (1981) theory that inflation uncertainty has a positive impact on the volatility of growth.

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