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Andreas Irmen and
Johanna Kuehnel

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Andreas Irmen[1]

University of Heidelberg, CEPR, London, and CESifo, Munich

Johanna Kuehnel[2]

University of Heidelberg

Abstract: We provide a comprehensive survey of the recent literature on the link between productive government expenditure and economic growth. Starting with the seminal paper of Robert Barro (1990) we show that an understanding of the core results of the ensuing contributions can be gained from the study of their respective Euler equations. We argue that the existing literature incorporates many relevant aspects, however, policy recommendations tend to hinge on several knife-edge assumptions. Therefore, future research ought to focus more on idea-based endogenous growth models to check the robustness of policy recommendations. Moreover, the inclusion of hitherto unexplored types of government expenditure, e. g., on the "rule of law", would be desirable.

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[1]Address: Department of Economics, University of Heidelberg, Grabengasse 14, D-69117 Heidelberg, Germany, airmen@uni-hd.de.

[2]Address: Department of Economics, University of Heidelberg, Grabengasse 14, D-69117 Heidelberg, Germany, johanna.kuehnel@awi.uni-heidelberg.de.

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Contents

1 Introduction

It is widely recognized that public expenditure on infrastructure such as roads, ports, or communication systems, public research spending and the provision of basic education and medical services raises the economic potential of an economy. At least since the influential study of Aschauer (1989) and the following discussion (see, de Haan and Romp (2007) for a recent survey of this empirical literature) it is argued that a rise in productive government activity increases output. Easterly and Rebelo (1993) and, more recently, Canning and Pedroni (2004) find evidence for long-run growth effects associated with public investment in infrastructure. In addition, many case studies highlight the growth-enhancing potential associated with such investments (see, e. g., OECD (2007)).

The purpose of this paper is to provide a critical survey on the recent theoretical literature that aims at the identification of possible links between productive government activity and long-run economic growth and the assessment of the resulting allocation in terms of welfare. To accomplish this, we have to focus on endogenous growth models where variations in fiscal policy parameters may have an effect on long-run growth.[1] To the best of our knowledge, Barro (1990) is the seminal paper in this field. It introduces government expenditure as a public good into the production function of individual firms. In this way the rate of return to private capital increases which stimulates private investment and growth.

We show that the ensuing literature is able to extend Barro's analysis to incorporate many relevant aspects that interact with the effect of public services on economic growth. They include adjustment costs, congestion effects, utility-enhancing public consumption services, endogenous labor supply both in closed and small open economies. We establish that the mechanics and the core results for each aspect can be gained from the study of the respective Euler equation. We use this approach to characterize the determinants of the equilibrium growth rate and to analyze the role of fiscal policy measures on this growth rate. Moreover, we conduct a welfare analysis and derive the circumstances under which the welfare-maximizing allocation can be implemented.[2]

While Barro (1990) treated productive government expenditure as a flow variable, the paper by Futagami, Morita, and Shibata (1993) introduces the provision of productive

[1] For the study of various aspects of public expenditure in the neoclassical growth model, the interested reader is referred to Arrow and Kurz (1970), Aschauer (1988), Barro (1989), Baxter and King (1993), Fisher and Turnovsky (1995), or Fisher and Turnovsky (1998).

[2] Throughout we stick to a continuous-time framework with infinitely-lived dynasties. Moreover, we do not explicitly consider education and human capital formation as a government activity. This is at the heart of, e. g., Glomm and Ravikumar (1992) or, more recently, Gómez (2007). See Zagler and Dürnecker (2003) for a survey of this literature.

government services as a stock. At first sight, this approach is more appealing since services like public infrastructure are more realistically described as stocks. However, the advantage in terms of realism has a price in terms of analytical complexity. For instance, this approach usually entails complex transitional dynamics and the steady-state growth rate is no longer determined by the Euler equation alone. Nevertheless, we argue that the analysis of the balanced growth path in the stock case confirms most results that are obtained in the flow case. An important difference occurs in the welfare analysis. The fact that current public investments become only productive tomorrow tends to reduce the welfare-maximizing share of government investment. We show that these findings are robust in a setting where a flow and a stock of public services are provided simultaneously.

However, the stock approach allows to address new questions that cannot be raised in a flow context. We make this point with an analysis that introduces an additional productive use of government expenditure, namely the maintenance of the public capital stock.

Finally, we turn to more fundamental variations of the analytical framework and ask for the robustness of the policy implications derived so far. For a stochastic setting we conclude that the policy implications are similar in spite of the fact that precautionary savings drive a wedge between the goals of growth and welfare maximization. In contrast, we argue that the knife-edge assumption of constant returns to scale with respect to private and public capital is responsible for many findings. For instance, under increasing returns multiple equilibria are endemic. This complicates the policy implications since the effect of fiscal policy measures is now conditional on expectations. Similarly, if productive government services are provided in a non-scale model, they cease to have an effect on the steady-state growth rate.

In light of these findings, we conclude that future research ought to focus on a deeper understanding of the policy implications that matter in reality. Certainly, a focus on the analysis of productive government services on economic growth in idea-based endogenous growth models is likely to enhance our understanding of the relationship between productive government expenditure and economic growth.

The remainder of this paper is organized as follows. Section 2 sets out the basic analytical framework. In Section 3 we deal with the flow model and variants of this approach. Section 4 presents variants of the stock approach and compares them to the respective flow cases. Important extensions such as uncertainty, increasing returns and non-scale models are covered in Section 5. Section 6 concludes. The Appendix derives somewhat more complicated results that appear in Section 4.1.

2 The Basic Analytical Framework

Consider a closed economy in continuous time with many identical and competitive household-producers and a government. We denote per-household magnitudes by small letters, whereas capital letters represent aggregates. For instance, $k(t)$ is the private capital input of an individual firm at t, and $K(t)$ the economy's aggregate capital stock at t. Henceforth, we suppress the time argument whenever this does not cause confusion. We represent household-producers by the interval $[0, N]$, $N > 1$, such that $K = Nk$. The "number" of household-producers remains constant over time. The economy has one good that can be consumed or invested. At all t, prices are expressed in units of the contemporaneous output of this good.

Each producer has access to the per-period production function

$$y = f(k, g) = Ak^{1-\alpha}g^{\alpha}, \qquad 0 < \alpha < 1, \tag{2.1}$$

where y denotes firm output at t, $A > 0$ the time-invariant total factor productivity, and g the services derived by the firm from productive government activity at t. Private capital, k, has a positive but diminishing marginal product, and for simplicity does not depreciate.[3] The function f has constant returns to scale with respect to both inputs. The possibility of steady-state growth arises since government activity acts as a countervailing force on the diminishing marginal product of private capital. To keep the marginal and the average productivity of private capital constant, in a steady state k and g have to grow at the same rate. To simplify the exposition, we work with the Cobb-Douglas specification.

Household-producers are infinitely-lived and derive utility in each period from private consumption. Their intertemporal utility is

$$u = \int_0^\infty e^{-\rho t} \ln c \, dt, \tag{2.2}$$

where $\rho > 0$ is the instantaneous rate of time preference. For expositional convenience we stick to a logarithmic per-period utility function. Most of the results presented in what follows readily extend to more general per-period utility functions with a constant elasticity of intertemporal substitution different from unity. Each household-producer receives net

[3] Labor is not mentioned as a separate input in the production function. This is a valid shortcut if we interpret the profit of each firm as the wage income that is earned by labor. More precisely, we may admit to each household-producer an exogenous per-period labor endowment $\bar{l} = 1$ that is inelastically supplied and consider a production function $y = Ak^{1-\alpha}\left(\bar{l}g\right)^{\alpha}$. Marginal cost pricing of labor and a real wage consistent with firms hiring $\bar{l}$ determines the wage income equal to the profit income of firms that produce according to (2.1) without labor. This is an implication of Euler's law for linear-homogeneous functions.

output and determines how much to consume and how much to invest in private capital. Her flow budget constraint is

$$\dot{k} = (1 - \tau_y)\, f\,(k, g) - (1 + \tau_c)c - \tau, \tag{2.3}$$

where τ_y and τ_c denote time-invariant tax rates on income/output and consumption, and τ is a lump-sum tax. When choosing c and k to maximize her utility the individual household-producer takes the level of public services as given and disregards the possible impact of her decision on the amount of public services provided. Then, her intertemporal optimization leads to the Euler condition

$$\gamma_c \quad = \quad (1 - \tau_y)\frac{\partial f}{\partial k} - \rho, \tag{2.4}$$

i. e., the growth rate of consumption γ_c depends on the difference between the after-tax private marginal return on private capital and the rate of time preference. Throughout, we assume that the economy is sufficiently productive to sustain a strictly positive growth rate γ_c.

We denote G the aggregate amount of productive government activity at t from which individual firms derive the services g. Conceptually, G may be a flow or a stock variable. In the former case, government spending corresponds to the provision of public services that instantaneously affect the production technology of firms. In the latter case, today's government spending adds to the stock of public capital and affects the future production technology of firms. In any case, the government claims resources from household-producers and transforms them one-to-one into a productive input to which firms get access. We assume that the government's budget is balanced in all periods. Let Y and C denote aggregate output and consumption at t and define total tax receipts at t by $T \equiv \tau_y Y + \tau_c C + \tau N$. Then, the budget constraint at t is

$$G = T \quad \text{or} \quad \dot{G} = T \tag{2.5}$$

for the flow and the stock case, respectively. Throughout, we focus on tax-financed expenditure and disregard funding via public debt.

3 Productive Government Activity as a Flow

Along a steady-state growth path with all variables growing at a constant rate, government expenditure must be proportionate to the size of the economy. To comply with this requirement, we stipulate for all t that

$$G = \theta_G Y, \tag{3.1}$$

where $\theta_G \in (0,1)$ is a time-invariant constant measuring the fraction of current output that constitutes the current flow of productive government expenditure. If G includes public investment as well as government expenditure on public order and safety, on economic affairs, and on health and education, one finds for a sample of 19 OECD countries that the average θ_G over the time period 1995 to 2002 ranges between 10% and 20%.[4]

3.1 The Pure Public Good Case

Following Barro (1990), we first consider the case where government services are neither rival nor excludable. In this case, G is a pure public good and $g = G$ such that the production function (2.1) becomes

$$y = AG^{\alpha}k^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{3.2}$$

One may of think of G as government expenditure on basic education, the provision of medical services, or public research spending that increases the productivity of private inputs of all firms in the same manner.

Decentralized Equilibrium

Following the reasoning that led to the Euler equation (2.4) we find

$$\gamma_c \quad = \quad (1-\tau_y)(1-\alpha)A\left(\frac{G}{k}\right)^{\alpha} - \rho. \tag{3.3}$$

The ratio of government spending per unit of private capital consistent with condition (3.1), the aggregation $Y = Ny$, and the production function (3.2) is $G/k = (AN\theta_G)^{1/(1-\alpha)}$. Upon substitution of the latter in (3.3) we obtain

$$\gamma_c = (1-\tau_y)(1-\alpha)\left(AN^{\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{3.4}$$

At this stage, three remarks are in order. First, there are admissible values for τ_y, τ_c, τ, and θ_G that satisfy the budget constraint (2.5). Hence, given τ_c and τ, τ_y and θ_G that appear in (3.4) are not independent. Second, one can show that the economy immediately jumps onto its steady-state path along which all per-capita magnitudes grow at rate γ_c. Third,

[4] This finding is based on our own computations. We use data collected in UNdata (2008). Public investment corresponds to gross fixed capital formation of general government. Government expenditure on public order and safety, on economic affairs, and on health and education are subcategories of government final consumption expenditure. The sample includes Austria, Belgium, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Mexico, the Netherlands, Norway, Portugal, Sweden, the United Kingdom and the United States of America. This selection of OECD countries maximizes the number of countries and the length of the time period for which a full set of comparable annual data on the components of G mentioned above are available. The sample average of θ_G across countries and time is approximately 15%.

the equilibrium growth rate depends on the "number" of household-producers, i. e., there is a scale effect. The latter occurs since individual firm productivity depends on aggregate spending ($G = \theta_G Y$). Then, with more firms the externality is more pronounced.[5]

An interesting question is how the size of the government and the mode of funding government spending affects the economy's steady-state growth rate. A useful benchmark has $\tau_c = \tau = 0$ such that $\theta_G = \tau_y$. In this case, there is a growth-maximizing expenditure share equal to the output elasticity of government expenditure, $\theta_G^* = \alpha$.[6] Intuitively, it balances two opposing effects. A rise in θ_G increases the private marginal product of private capital and reduces its after-tax value through a necessary increase in the distortionary income tax. At θ_G^*, government expenditure satisfies the so-called *natural condition of productive efficiency*, i. e., the marginal contribution of government expenditure to aggregate output is one.[7] The steady-state growth rate may be further increased if a strictly positive consumption or lump-sum tax is levied. In the present context, a consumption tax acts like a lump-sum tax and both may be used to reduce the distortionary income tax. However, there is little reason why the steady-state growth rate should be arbitrarily large since faster economic growth has a cost in terms of foregone consumption. To assess the desirability of a given consumption growth rate we have to compare it to the allocation chosen by an omniscient social planner.

Pareto Efficiency

Contrary to the household-producer, the social planner knows that - given θ_G - the choice of k affects the level of government expenditure G through condition (3.1) and $Y = Ny$. Hence, he perceives the production function of the representative household-producer as

$$y = (AN^\alpha \theta_G^\alpha)^{\frac{1}{1-\alpha}}\, k. \tag{3.5}$$

The aggregate resource constraint is $N\dot{k} = Ny - G - Nc$. It results as the sum of all individual flow budget constraints (2.3) in conjunction with the government's budget

[5] To eliminate the scale effect one may assume that the government service is not excludable but rival such that each producer receives a proportionate share of government services, i. e., $g = G/N$. In this case, the economy's steady-state growth rate is $\gamma_c = (1-\tau_y)(1-\alpha)\,(A\theta_G^\alpha)^{\frac{1}{1-\alpha}} - \rho$ and is independent of N. We shall get back to this case in Section 3.3 where we discuss different forms of congestion.

[6] For more general production functions $f(k, g)$ with constant returns to scale in its inputs the growth-maximizing expenditure share remains equal to the respective output elasticity. This elasticity, however, need not be constant but may vary with G and other parameters. This generalization may prevent closed-form solutions (see, e. g., Ott and Turnovsky (2006) for a discussion).

[7] To grasp the natural condition of productive efficiency consider a coal mine that uses its coal as an input. Then, what is the output-maximizing amount of the coal input? Intuitively, as long as an additional unit of coal raises output by more than one unit it will be used; if its marginal product is smaller, it will not. The quantity that maximizes output obtains when the marginal product of coal in its production is one. In the present context, we have from equation (3.2) with $Y = Ny = ANG^\alpha k^{1-\alpha}$: $dY/dG = \alpha\,(Y/G) = \alpha/\theta_G^* = 1$.

constraint and (3.1). Expressed in per-household terms, this is

$$\dot{k} = (1 - \theta_G)y - c. \tag{3.6}$$

Throughout, we shall refer to an allocation as *constrained Pareto-efficient* if the planner takes the share of government expenditure θ_G as a given constant. The unconstrained or, in short, the *Pareto-efficient* allocation is the one obtained when the planner chooses θ_G optimally.

Here, the constrained Pareto-efficient allocation obtains from the maximization of u given by (2.2) with respect to c and k subject to (3.6). The corresponding Euler condition delivers the steady-state growth rate of all per-household magnitudes and is given by

$$\gamma_c^P = (1 - \theta_G)\left(AN^\alpha \theta_G^\alpha\right)^{\frac{1}{1-\alpha}} - \rho. \tag{3.7}$$

The first term on the right-hand side is the social marginal return on private capital. It does not coincide with the after-tax private marginal product that matters in (3.4). The comparison of the equilibrium to the planner's growth rate reveals that

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1 - \tau_y)(1 - \alpha) = 1 - \theta_G. \tag{3.8}$$

These growth rates generically differ for two reasons. First, in equilibrium intertemporal prices are distorted due to the income tax. If the government sets $\tau_y = 0$ to eliminate this distortion and finances its expenditure via lump-sum taxes these growth rates may still differ as θ_G need not be equal to α. This reflects the second difference. The planner internalizes the externality associated with the provision of the public good, i. e., when choosing c and k he accounts for condition (3.1).

If we extend the planner's choice set and allow him to determine the size of the government in addition to c and k, one finds that he chooses $\theta_G^P = \alpha$. The Pareto-efficient growth rate is then given by γ_c^P of (3.8) with $\theta_G = \alpha$. As a consequence, the equilibrium and the Pareto-efficient allocation coincide if $\tau_y = 0$ and θ_G is chosen optimally.

3.2 Productive Government Expenditure and Adjustment Costs

Often the productive use of new private capital requires adjustment costs. Examples include costs for the installation of equipment or the schooling of employees. Adjustment costs increase the effective costs of private investment and may therefore discourage the accumulation of private capital. Here, we introduce this feature into the pure public good framework of the previous section.

Following Turnovsky (1996a) we assume that productive government expenditure reduces adjustment costs. For instance, due to a better road network the setup costs of a new factory may be lower. We capture this feature with an adjustment cost function per unit of investment given by $\phi(\theta_G)\, i/(2k)$, where i denotes investment per household-producer. A higher share of government activity reduces adjustment costs, though at a declining rate, i. e., $\phi' < 0 < \phi''$. As in Hayashi (1982), we assume that adjustment costs are proportional to the rate of investment per unit of installed capital and not to the absolute level of investment. Accordingly, the investment cost function is

$$\varphi(i, k, \phi(\theta_G)) \equiv \left(1 + \frac{\phi(\theta_G)}{2}\frac{i}{k}\right) i. \tag{3.9}$$

Decentralized Equilibrium

Individual household-producers choose a plan (c, k, i) for each t to maximize u of (2.2) subject to the constraints

$$i = \dot{k} \quad \text{and} \tag{3.10}$$

$$(1 - \tau_y)\, AG^{\alpha}k^{1-\alpha} - \tau = (1 + \tau_c)c + \varphi(i, k, \phi(\theta_G)), \tag{3.11}$$

where the latter equalizes disposable income to consumption and investment outlays.

The resulting optimality condition with respect to i reveals that the current-value shadow price of installed capital in units of current output is equal to the marginal investment costs, i. e.,

$$q = 1 + \phi(\theta_G)\frac{i}{k}. \tag{3.12}$$

Hence, for an investing firm $(i > 0)$ the value of installed capital exceeds unity. With (3.10) it follows that the steady-state growth rate of private capital is

$$\gamma_k = \frac{q - 1}{\phi(\theta_G)}. \tag{3.13}$$

The Euler equation is now given by

$$\gamma_c = \frac{(1 - \tau_y)(1 - \alpha)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}}}{q} + \frac{\dot{q}}{q} + \frac{(q - 1)^2}{2q\phi(\theta_G)} - \rho. \tag{3.14}$$

The first three terms on the right-hand side represent the rate of return on acquiring a unit of private capital at price q. The first term denotes the after-tax private marginal return on private capital deflated by the cost of capital q. The second term is the rate of capital gain. The third term reflects the marginal reduction in adjustment costs when k increases for given i deflated by q. In the absence of adjustment costs $q = 1$ for all t and (3.14) reduces to (3.4).

In the steady state, per-household magnitudes such as c, k, and i grow at the same rate; from (3.12) we also have $\dot{q} = 0$. Using (3.12) and (3.13) in the Euler condition (3.14) delivers the steady-state growth rate implicitly as

$$\frac{\phi(\theta_G)}{2}\gamma_c^2 + \left[1 + \rho\phi(\theta_G)\right]\gamma_c = (1 - \tau_y)(1 - \alpha)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{3.15}$$

Hence, the right-hand side of (3.15) coincides with the equilibrium growth rate of (3.4) where adjustment costs are absent. However, since the left-hand side of (3.15) increases faster than proportionately in γ_c, the resulting steady-state growth rate must be smaller with than without adjustment costs.

Turning to the effect of productive government expenditure on the steady-state growth rate for the benchmark scenario with full income tax funding ($\theta_G = \tau_y$) we find

$$\frac{d\gamma_c}{d\theta_G} = \frac{-\gamma_c \phi'(\theta_G)\left(\frac{\gamma_c}{2} + \rho\right) + \alpha(1-\theta_G)\frac{y}{k\theta_G} - (1-\alpha)\frac{y}{k}}{1 + \phi(\theta_G)(\gamma_c + \rho)}. \tag{3.16}$$

The latter highlights three channels through which government activity affects the growth rate. First, the reduction in adjustment costs ($\phi' < 0$) increases the growth rate. The second and the third channel matter in the same way as in the scenario without adjustment costs: on the one hand, productive government expenditure enhances the productivity of the existing capital stock, on the other hand, the government must balance its budget which brings about a rise in the distortionary income tax rate.

Observe that a growth-maximizing expenditure share $\theta_G^* \in (0,1)$ may exist. It must be strictly greater than in the world without adjustment costs since $d\gamma_c/d\theta_G|_{\theta_G=\alpha} > 0$. If government expenditure is fully funded by a non-distortionary lump-sum tax the third channel in (3.16) vanishes such that an increase in government spending unambiguously raises the growth rate.

Pareto Efficiency

The social planner internalizes (3.1) and the equilibrium condition $K = Nk$. He maximizes utility (2.2) subject to the resource constraint $(1-\theta_G)\,y = c + \varphi(i, k, \phi(\theta_G))$ and $i = \dot{k}$, where y is given by (3.5). Following the steps that led to the implicit statement of the equilibrium growth rate in (3.15), we obtain here

$$\frac{\phi(\theta_G)}{2}\left(\gamma_c^P\right)^2 + \left[1 + \phi(\theta_G)\rho\right]\gamma_c^P = (1-\theta_G)\left(A\theta_G^\alpha N^\alpha\right)^{\frac{1}{1-\alpha}} - \rho. \tag{3.17}$$

The latter generalizes (3.7) to the case with adjustment costs. Again the left-hand side is strictly convex in γ_c^P such that the constrained optimal growth rate is smaller under adjustment costs.

If we allow the social planner to determine the size of θ_G optimally, the welfare-maximizing share of government expenditure, θ_G^P, is greater than α, thus exceeding its level without adjustment costs. Intuitively, the possibility to reduce adjustment costs provides an additional incentive for the government to expand its activity relative to the size of the economy.

The comparison of the equilibrium to the constrained optimal growth rate reveals that both rates are the same if the right-hand sides of (3.15) and (3.17) take on the same value.

From (3.8) we know that this is the case whenever $(1-\tau_y)(1-\alpha) = 1-\theta_G$. Interestingly, adjustment costs alter the implications of this condition for the optimal tax policy. For instance, if government expenditure is fully financed via lump-sum taxes ($\tau_y = 0$), then the Pareto-efficient growth rate cannot be implemented since $\theta_G^P > \alpha$. The reason is that a higher θ_G^P does not only internalize the externality associated with the pure public good but also reflects the planner's incentive to reduce adjustment costs. Therefore, at θ_G^P the equilibrium incentives to invest are too pronounced relative to the efficient growth rate. Accordingly, a strictly positive income tax $\tau_y^P = \left(\theta_G^P - \alpha\right)/(1-\alpha) > 0$ is needed to support the Pareto-efficient allocation.

3.3 Public Goods Subject to Congestion

Often, the services derived from the provision of a public good are subject to congestion. Congestion effects arise if public goods are partially rival, i. e., their use as a productive input by one firm diminishes their usefulness to other firms. Examples include road infrastructure or police and fire protection.

Two forms of congestion can be distinguished, relative and aggregate (absolute) congestion. In the former case, the level of services derived by an individual firm depends on its size relative to the aggregate of firms. We refer to aggregate congestion if the level of services received by the individual firm is decreasing in the aggregate usage. As noted by Eicher and Turnovsky (2000), p. 344, highway usage is an example of the former and police protection an example of the latter:

> "Unless an individual drives his car, he derives no service from a publicly provided highway, and in general the services he derives depend upon his own usage relative to that of others in the economy, as total usage contributes to congestion. Police protection may serve as an example of absolute congestion: in principle, people always enjoy this service, independent of their own actions, though the amount of service they may actually derive varies inversely with aggregate activity and the demands this places on the limited resources devoted to this public service."

To study relative congestion, we use the ratio of individual to aggregate private capital, k/K, to measure the size of an individual firm relative to the economy. Then, the productive services that a firm derives from public expenditure G is

$$g = G\left(\frac{k}{K}\right)^{1-\sigma_G}, \tag{3.18}$$

where $\sigma_G \in [0,1]$ parameterizes the degree of relative congestion associated with the public good G. This specification includes the pure public good case (without congestion) for $\sigma_G = 1$. As σ_G declines, congestion becomes more pronounced. Yet, as long as $\sigma_G \in (0, 1)$, the government services derived by a firm of size k increases if G and K grow at the same rate. Barro and Sala-í-Martin (1992) analyze the case where $\sigma_G = 0$. Then, g increases only if G grows faster than K. The latter case is called *proportional congestion* (Turnovsky (2000b), p. 618). As in equilibrium $K = Nk$, the public good is then rival yet not excludable and the individual firm receives its proportionate share of services $g = G/N$.

One specification the literature uses to capture aggregate congestion is $g = GK^{\sigma_G - 1}$, $\sigma_G \in [0, 1]$, i. e., government services are independent of firm size. With this specification, the firms' production function ceases to exhibit constant returns to scale in private and public capital. Therefore, steady-state growth can only arise under additional restrictive conditions. To avoid these complications, we restrict attention to the case of relative congestion with and without excludability.[8] If a public good is excludable, then the government can identify the user and charge an access fee.

3.3.1 Relative Congestion Without Excludability

Under relative congestion, we obtain the production function of the individual firm from (2.1) and (3.18) as

$$y = A\left(\frac{G}{K}\right)^{\alpha}\left(\frac{k}{K}\right)^{-\sigma_G\,\alpha} k. \tag{3.19}$$

Decentralized Equilibrium

Individual firms believe that a rise in k increases their benefit from the provision of public services and disregard the impact of their investment decision on G and K. Applying the reasoning that led to the Euler equation (3.4) and taking into account that $G/k = \left(A\theta_G N^{1-\alpha(1-\sigma_G)}\right)^{1/1-\alpha}$ we find

$$\gamma_c = (1-\tau_y)(1-\sigma_G\alpha)\left(AN^{\sigma_G\alpha}\theta_G^{\alpha}\right)^{\frac{1}{1-\alpha}} - \rho. \tag{3.20}$$

Again, the first term on the right-hand side is the after-tax marginal private return on private capital. An increase in the degree of congestion, i. e., a decline in σ_G, has two effects on γ_c: On the one hand, it augments the output elasticity of private capital, $1 - \sigma_G\alpha$. On the other hand, it weakens the scale effect through $N^{\sigma_G\alpha}$. Which effect dominates depends on the number of household-producers and σ_G.

[8] See, e. g., Glomm and Ravikumar (1994) for a discrete-time model with absolute congestion. Their setup has a one period lag between the collection of taxes and the conversion of these revenues into public services. Hence, methodologically this study belongs to the "stock case" to which we turn in Section 4. Ott and Soretz (2007) argue that relative congestion of productive government activity may also be important for the spatial distribution of economic activity.

As in the pure public good case, the growth-maximizing share of government expenditure θ_G^* for the benchmark scenario with full income tax financing is equal to α.

Pareto Efficiency

The social planner is aware of the negative externality that the choice of k by an individual firm exerts on the production technology of all other firms via the implied increase in the aggregate capital stock K. He also knows that in a symmetric configuration no firm can gain an advantage from the provision of public services by raising its capital stock. Since all firms are identical, no firm can increase its size relative to other firms and/or the economy.

In other words, the planner internalizes the equilibrium condition $K = Nk$ in (3.18) which then reduces to $g = GN^{\sigma_G - 1}$. As a consequence, with (3.1) the relevant production function is $y = (AN^{\sigma_G\,\alpha}\theta_G^{\alpha})^{1/1-\alpha}\,k$ and the constrained efficient growth rate of consumption becomes

$$\gamma_c^P = (1-\theta_G)\,(AN^{\sigma_G\,\alpha}\theta_G^{\alpha})^{\frac{1}{1-\alpha}} - \rho. \tag{3.21}$$

The comparison of the equilibrium growth rate (3.20) to the constrained optimal growth rate (3.21) gives

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_y)(1-\sigma_G\,\alpha) = 1-\theta_G. \tag{3.22}$$

With $\sigma_G < 1$ the latter two equations generalize (3.7) and (3.8) to the case of congestion. With congestion, the equilibrium growth rate may again be too high relative to the efficient one. To see this, consider the case where θ_G is chosen optimally, i. e. $\theta_G^P = \alpha$. Then, if government expenditure is entirely financed by lump-sum taxes, i. e., $\tau_y = 0$, it holds that $\gamma_c > \gamma_c^P$. Intuitively, congestion drives a wedge between the private and the social marginal return to private capital and induces an incentive to over-accumulate private capital in the decentralized equilibrium. From (3.22) we derive the income tax rate for an optimally chosen share of government expenditure as

$$\tau_y^P = \frac{\alpha\,(1-\sigma_G)}{1-\sigma_G\alpha}, \quad \text{with} \quad \frac{d\tau_y^P}{d\sigma_G} < 0. \tag{3.23}$$

An income tax rate τ_y^P eliminates this wedge and implements the Pareto-efficient allocation. Clearly, τ_y^P increases the stronger the degree of congestion. In the extreme case of proportional congestion all government expenditure should be financed via income taxes, i. e. $\tau_y^P = \theta_G^P = \alpha$.

3.3.2 Relative Congestion With Excludability

Some public services subject to congestion are excludable. This means that a potential user of the service can be identified and charged a user fee. Examples include highways, bridges, universities, or schools.

Ott and Turnovsky (2006) extend the previous setup and introduce a second public service that is excludable. The modified production function of individual firms is

$$y = f(k, g, e) = Ag^{\alpha} e^{\beta} k^{1-\alpha-\beta}, \qquad 0 < \alpha, \beta < 1, \tag{3.24}$$

where e is the benefit derived by the firm from the excludable public service. Just as the non-excludable public service G, the excludable one is subject to relative congestion such that

$$e = E \left(\frac{k}{K} \right)^{1-\sigma_E}; \tag{3.25}$$

here, E is the total amount of the excludable public service supplied by the government, and $\sigma_E \in [0, 1]$ measures the degree of relative congestion. Using (3.25) and (3.18) in (3.24) gives the production function as perceived by the individual firm

$$y = A \left(\frac{G}{K} \right)^{\alpha} \left(\frac{k}{K} \right)^{-\alpha\sigma_G} \left(\frac{E}{K} \right)^{\beta} \left(\frac{k}{K} \right)^{-\beta\sigma_E} k. \tag{3.26}$$

For the government, the key difference between the provision of G and E is that the former must be financed through taxes whereas the latter can be financed through a fee paid by the individual user. Denote p this fee per unit of E. Then, the expression for the balanced government budget (2.5) becomes $G + E = \tau_y Y + \tau_c C + \tau N + pEN$. Similar to condition (3.1) for G, we assume that the provision of E is proportionate to the size of the economy, i. e., $E = \theta_E Y$ for all t.

Decentralized Equilibrium

For the individual household-producer, the new element is that besides c and k, she also determines in each period her demand for the excludable public service. The associated expenditure pE^d must be added to the flow budget constraint (2.3) which modifies to $\dot{k} = (1 - \tau_y) f(k, g, e) - (1 + \tau_c)c - \tau - pE^d$.

Since the choice of E^d does not affect utility directly, it is chosen to maximize the right-hand side of the flow budget constraint. The associated optimality condition equates the marginal after-tax product of the excludable input to its marginal cost, i. e.,

$$(1 - \tau_y) \frac{\partial y}{\partial E^d} = (1 - \tau_y) \beta \frac{y}{E^d} = p. \tag{3.27}$$

The latter delivers the demand of each household-producer, $E^d(p)$. Since E is a public good, in equilibrium we have $E = E^d(p)$. Together with the proportionality constraint $E = \theta_E Ny$, we obtain from (3.27) the equilibrium value of p as

$$p = \frac{(1 - \tau_y)\beta}{\theta_E N}. \tag{3.28}$$

Intuitively, the equilibrium user fee declines with the total number of users.

Applying the same reasoning that led to the Euler equation (3.20) delivers the equilibrium growth rate

$$\gamma_c = (1-\tau_y)(1-\alpha\sigma_G-\beta\sigma_E)\left(AN^{\sigma_G\,\alpha+\sigma_E\,\beta}\theta_G^{\alpha}\theta_E^{\beta}\right)^{\frac{1}{1-\alpha-\beta}}-\rho, \tag{3.29}$$

which generalizes (3.20) of the non-excludable public input case to $\beta > 0$. Since firms neglect the congestive consequences of their own choice of the private capital input on the aggregate economy, they continue to overestimate the before-tax marginal product of capital.

Finally, consider the growth-maximizing government expenditure shares for the benchmark scenario where the provision of G is fully financed through income taxes, i. e, $\theta_G = \tau_y$. With two public goods and a user fee given by (3.28) the two shares θ_G and θ_E are linked by the government budget such that $\theta_E = \beta\,(1-\theta_G)$. Using this condition, we obtain $\theta_G^* = \alpha$ and $\theta_E^* = \beta\,(1-\alpha)$.

Pareto Efficiency

The social planner internalizes congestion effects, i. e., he considers (3.26) in conjunction with $K = Nk$ and the proportionality conditions $Y = G/\theta_G = E/\theta_E$. Then, $y = \left(AN^{\sigma_G\,\alpha+\sigma_E\,\beta}\theta_G^{\alpha}\theta_E^{\beta}\right)^{1/1-\alpha-\beta} k$ and the resource constraint is $\dot{k} = (1-\theta_G-\theta_E)y - c$. The constrained efficient growth rate obtains as

$$\gamma_c^P = (1-\theta_G-\theta_E)\left(AN^{\sigma_G\,\alpha+\sigma_E\,\beta}\theta_G^{\alpha}\theta_E^{\beta}\right)^{\frac{1}{1-\alpha-\beta}}-\rho, \tag{3.30}$$

the first term on the right-hand side denoting the social marginal return on private capital.

If the social planner is allowed to choose θ_G and θ_E optimally, he picks $\theta_G^P = \alpha$ and $\theta_E^P = \beta$.[9]

The comparison of the equilibrium to the constrained optimal growth rate reveals that

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_y)(1-\sigma_G\alpha-\sigma_E\beta) = 1-\theta_G-\theta_E. \tag{3.31}$$

The latter has interesting consequences for the budgeting of government services. To see this, consider the case where $\theta_G^P = \alpha$ and $\theta_E^P = \beta$. Then, (3.28) and (3.31) deliver the following pair (τ_y^P, p^P) that implements the efficient allocation

$$\tau_y^P = \frac{(1-\sigma_G)\alpha + (1-\sigma_E)\beta}{1-\sigma_G\alpha-\sigma_E\beta} \quad \text{and} \quad p^P = \frac{1}{N}\left(\frac{1-\alpha-\beta}{1-\sigma_G\alpha-\sigma_E\beta}\right). \tag{3.32}$$

[9] Again, this result can be linked to the natural condition of productive efficiency. We obtain from equation (3.26) with $K = Nk$ and $Y = Ny$ that $dY/dG = \alpha\,(Y/G) = \alpha/\theta_G^P = 1$ and $dY/dE = \beta\,(Y/E) = \beta/\theta_E^P = 1$. Hence, the marginal product of both government services provided out of current production is one.

To satisfy the government's budget constraint at (τ_y^P, p^P) a residual lump-sum tax or subsidy may be necessary.

As a benchmark, consider the case where $\sigma_G = \sigma_E = 1$ such that both public services are congestion-free. Then, $\tau_y^P = 0$ and $p^P = 1/N$, i. e., there is no distortion of intertemporal prices and each firm's demand for the excludable public service satisfies the natural efficiency condition $\partial y/\partial E^d = p^P = 1/N$. In this case, the user fee fully finances the provision of E. However, the provision of G must be financed through some lump-sum tax to guarantee a balanced budget.

In the presence of congestion, $\tau_y^P > 0$ is necessary to correct for the congestion externalities. However, as τ_y^P increases the price of the excludable service must fall since its after-tax marginal product declines. Then, the provision of E requires cross-subsidization.

3.4 Public Consumption Services

Many publicly provided services matter for an economy because they directly enhance the utility of households without affecting technology. Examples include cultural and recreational public services such as museums, public parks, or public social events like fireworks. To study the role of such public consumption services, we extend the analysis of the pure public good case of Section 3.1 and add a non-excludable service that enters the utility function. This service is subject to absolute congestion. With these properties, our analysis combines the framework of Barro (1990) and Turnovsky (1996c).[10]

The household's intertemporal utility is now

$$u = \int_0^\infty e^{-\rho t} \left(\ln c + b_h \ln h \right) dt, \tag{3.33}$$

where h is the service the individual household derives from the public consumption good, and $b_h \geq 0$ measures the relative weight of this form of consumption. For simplicity, the per-period utility is separable in c and h.

The public consumption good is subject to aggregate congestion in total output such that the service, h, derived by each household falls short of the aggregate service, H, provided by the government. More precisely, we follow Turnovsky (1996c) and stipulate

$$h = H^{\sigma_H} \left(\frac{H}{Y} \right)^{1-\sigma_H}; \tag{3.34}$$

[10] Cazzavillan (1996) studies the role of a public good that simultaneously affects per-period utility and the production function of the representative household-producer. Under a more general utility function that allows for increasing returns in the consumption externality of public expenditure, he shows that local indeterminacy and endogenous stochastic fluctuations may arise.

here, $\sigma_H \in [0,1]$ measures the degree of aggregate congestion with $\sigma_H = 1$ and $\sigma_H = 0$ capturing the special cases of a pure public good and of proportional congestion, respectively.

On the production side, we maintain the production function of equation (3.2). On the government side, we need to add H as government expenditure such that a balanced budget requires $G + H = \tau_y Y + \tau_c C + \tau N$. As in the previous sections, we tie the size of H to the size of the economy: $H = \theta_H Y$.

Decentralized Equilibrium

The individual household-producer behaves as in Section 3.1. Since H is non-excludable, there is no optimization with respect to h. Moreover, when choosing k, she disregards the link between k, aggregate output Y, and h that materializes under congestion.

As a result, the expression of the consumption growth rate in equilibrium is again given by (3.4).[11] However, if at least some part of H is funded via the distortionary income tax, then, the level of γ_c that satisfies the government's budget constraint is smaller. To see this, consider the benchmark where $G + H = \tau_y Y$. Then, $\tau_y = \theta_G + \theta_H$ such that

$$\gamma_c = (1 - \theta_G - \theta_H)(1 - \alpha)\left(AN^\alpha \theta_G^\alpha\right)^{\frac{1}{1-\alpha}} - \rho. \tag{3.35}$$

Since the government channels additional resources into non-productive uses, the latter falls short of (3.4) with $\tau_y = \theta_G$. For the same reason, the growth-maximizing expenditure share of the productive government services $\theta_G^* = \alpha(1 - \theta_H)$, declines in θ_H. Further, in this case the growth-maximizing share of public consumption services is zero.

Pareto Efficiency

The omniscient planner considers the individual production function as in (3.5). The resource constraint is $\dot{k} = (1 - \theta_G - \theta_H)y - c$. The key new element appears in the per-period utility function. The planner knows that the congestion effect of equation (3.34), the proportionality requirement, $H = \theta_H Y$, and the aggregation $Y = Ny$ imply $h = \theta_H (Ny)^{\sigma_H}$. Hence via (3.5), the choice of k directly affects per-period utility for $\sigma_H > 0$. The resulting constrained efficient steady-state growth rate is

$$\gamma_c^P = (1 - \theta_G - \theta_H)\left(AN^\alpha \theta_G^\alpha\right)^{\frac{1}{1-\alpha}} - \frac{\rho}{1 + b_h \sigma_H}. \tag{3.36}$$

The first term on the right-hand side is the social marginal return on private capital. The second term is the social rate of time preference. The provision of the non-productive

[11] This result hinges to some extent on the separability of c and h in the per-period utility function. If the marginal utility of c depends on h, then the household's willingness to postpone consumption depends on the growth rate of h. In a steady-state with congestion, the latter need not coincide with the steady-state growth rate of all other per-capita magnitudes. We leave a more detailed study of the impact of the interaction between c and h on the steady-state growth rate for future research.

public service reduces this rate. Intuitively, the presence of b_h captures the fact that a higher capital stock tomorrow raises the level of h, and hence tomorrow's utility. This effect is stronger the smaller the congestion effect.[12]

The equilibrium and the planner's growth rate coincide if and only if

$$\gamma_c = \gamma_c^P \Leftrightarrow (AN^\alpha \theta_G^\alpha)^{\frac{1}{1-\alpha}} \left[(1-\tau_y)(1-\alpha) - (1-\theta_G-\theta_H)\right] = \rho \left[1 - \frac{1}{1+b_h\sigma_H}\right]. \quad (3.37)$$

The term in brackets on the left-hand side reflects the possible deviation of the private from the social marginal rate of return on private capital. The gap between these rates depends on the way government finances its expenditure. A novelty compared to the pure public good case of Section 3.1 is the deviation of the private and the social rate of time preference that appears on the right-hand side.

We may expand the planner's choice set and allow him to determine the size of θ_G and θ_H. Then, the efficient pair $\left(\theta_G^P, \theta_H^P\right)$ satisfies the following optimality conditions

$$\theta_G^P = \alpha \left[\theta_H^P (\sigma_H - 1) + 1\right], \quad (3.38)$$

$$\theta_H^P = \frac{b_h \rho}{1+b_h\sigma_H} \left[AN^\alpha \left(\theta_G^P\right)^\alpha\right]^{\frac{-1}{1-\alpha}}. \quad (3.39)$$

Assume that the pair $\left(\theta_G^P, \theta_H^P\right)$ is unique in $[0,1]^2$ such that equations (3.38) and (3.39) intersect only once as depicted in Figure 1.

Intersection A corresponds to the case where $b_h = 0$ and $\sigma_H \in [0,1]$. Then, there is no utility associated with h. Hence, independently of the degree of congestion, the planner chooses $\theta_H^P = 0$ and $\theta_G^P = \alpha$ and the optimal allocation coincides with the one of Section 3.1. Case B has $b_h > 0$ and $\sigma_H \in (0,1)$. Here, $\theta_H^P > 0$ and $0 < \theta_G^P < \alpha$. Moreover,

$$\frac{d\theta_G^P}{db_h} < 0 \quad \text{and} \quad \frac{d\theta_H^P}{db_h} > 0, \quad (3.40)$$

$$\frac{d\theta_G^P}{d\sigma_H} > 0 \quad \text{and} \quad \frac{d\theta_H^P}{d\sigma_H} < 0. \quad (3.41)$$

Since $b_h > 0$, the planner is ready to provide public consumption services according to the optimality condition $\partial u/\partial H = \partial u/\partial c$. As a consequence, the relative size of G falls. To grasp the effect of congestion, recall that the planner is aware of the positive effect of θ_G on h in the utility function. This effect is more pronounced the lower the degree of congestion. Hence, a rise in σ_H increases θ_G^P. In the limit $\sigma_H \to 1$, H is a pure public good and $\theta_G^P \to \alpha$ as shown as intersection C in the Figure 1. In any case, the welfare-maximizing share of public consumption services is positive. Hence, the provision

[12] In the special case of proportional congestion, where $\sigma_H = 0$, the effect of k on h disappears because (3.34) in conjunction with $H = \theta_H Y$ implies $h = \theta_H$.

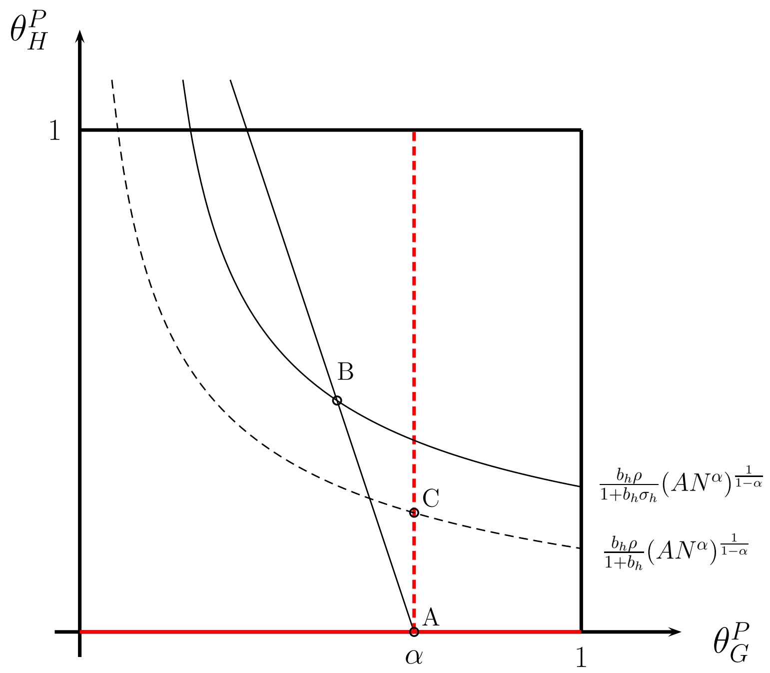

Figure 1: Efficient pair $\left(\theta_G^P, \theta_H^P\right)$

of public consumption services may introduce a wedge between the goals of growth and welfare maximization.[13]

The income tax rate that implements the Pareto-efficient allocation is found to be $\tau_y^P = (1 - \sigma_H)\,\theta_H^P$. Hence, without the congestion externality, i. e., $\sigma_H = 1$, no income tax is needed to implement the Pareto-efficient allocation.

3.5 Endogenous Labor Supply

This section incorporates the labor-leisure decision, i. e., individual labor supply becomes endogenous. In this context, a consumption tax as well as a tax on labor income are distortionary since they affect the trade-off between consumption and leisure. Contrary to the analysis of Section 3.4, public consumption expenditure turns out to have a positive effect on the equilibrium growth rate. We develop an intuition for this result following the presentation of Turnovsky (2000a).

The representative agent has a per-period time endowment equal to one and allocates the fraction $l \in (0,1)$ to leisure and $(1-l)$ to work. The per-period utility function takes the positive utility of leisure into account. More precisely, we stipulate

$$u \;=\; \int_0^\infty [\ln c + b_h \,\ln h + b_l \,\ln l]\; e^{-\rho t}\, dt, \quad b_h, b_l \geq 0. \tag{3.42}$$

[13]Park and Philippopoulos (2002) confirm this result in a setting that allows for a different set of second-best optimal policies.

Since the focus is on the role of labor supply, we abstract from congestion effects associated with the provision of public consumption services, i. e., $\sigma_H = 1$ in (3.34) such that $h = H$.

On the production side, we incorporate labor as a productive input and generalize the production function of (3.2) to

$$y = A\,[G\,(1-l)]^{\alpha}\,k^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{3.43}$$

Hence, there are constant returns to scale both with respect to private capital and labor, and with respect to public and private capital. The former implies zero profits in a competitive environment whereas the latter allows for steady-state growth of labor productivity.

On the government side, we split the proportional income tax τ_y into a tax on wage income at rate τ_w and a tax on capital income at rate τ_r. With w and r denoting the real wage and the real rate of return on private capital, the balanced government budget becomes $G + H = \tau_w w(1-l)N + \tau_r rK + \tau_c C + \tau N$.

Decentralized Equilibrium

Household-producers choose a plan (c, l, k) for each t such that (3.42) is maximized subject to the budget constraint $\dot{k} = (1-\tau_w)w(1-l) + (1-\tau_r)rk - (1+\tau_c)c - \tau$.

Following the steps that led to Euler equation (3.4) we obtain

$$\gamma_c = (1-\tau_r)(1-\alpha)\,(AN^{\alpha}\theta_G^{\alpha})^{\frac{1}{1-\alpha}}\,(1-l)^{\frac{\alpha}{1-\alpha}} - \rho. \tag{3.44}$$

Observe that l appears as a determinant of γ_c. Intuitively, since labor and capital are complements in the production function (3.43), more leisure reduces the marginal rate of return on private capital. Moreover, l is a choice variable that needs to be pinned down.

To find a second condition that determines the level of leisure consistent with steady-state growth, consider the product market equilibrium condition that coincides with the economy's resource constraint $\dot{k} = (1-\theta_G-\theta_H)\,y - c$. Expressing the latter in terms of the growth rate of private capital and using the static optimality condition for the consumption-leisure decision[14] delivers

$$\gamma_k \;=\; \left[(1-\theta_G-\theta_H) - \left(\frac{1-\tau_w}{1+\tau_c}\right)\frac{\alpha}{b_l}\left(\frac{l}{1-l}\right)\right](AN^{\alpha}\theta_G^{\alpha})^{\frac{1}{1-\alpha}}\,(1-l)^{\frac{\alpha}{1-\alpha}}. \tag{3.45}$$

A steady state needs $\gamma_c = \gamma_k$. If this requirement gives rise to a unique and strictly positive steady-state growth rate, then there is a time-invariant level of steady-state leisure depending on the policy variables $\theta_G, \theta_H, \tau_r, \tau_w, \tau_c$.

[14]The latter condition requires the marginal rate of substitution between leisure and consumption to equal the relative price of both goods, i. e., $b_l c/l = w(1-\tau_w)/(1+\tau_c)$. Marginal cost pricing of labor gives $w = \alpha y/(1-l)$. Using both equations, we can determine the ratio c/y, which is then used to derive (3.45).

Turnovsky (2000a) shows that a rise in either public consumption services ($\theta_H \uparrow$) or in public productive services ($\theta_G \uparrow$) financed by a lump-sum tax increases the steady-state growth rate. As to θ_H, this is the result of two opposing forces. For a given labor supply, the growth rate declines since the government claims additional resources. However, households increase their labor supply to make up for this negative income effect. Overall the steady-state growth rate increases due to greater employment. These effects do not materialize when labor supply is inelastic as in Section 3.4. In such a setting a lump-sum financed increase in θ_H has no impact on steady-state growth.

The same two forces also operate in the case of an increase in θ_G. In addition, there is a third effect since a higher θ_G raises the equilibrium wage and, hence, the labor supply. As a result, the steady-state growth rate increases further such that $\partial\gamma_c/\partial\theta_G > \partial\gamma_c/\partial\theta_H > 0$.

If the lump-sum tax is accompanied by a consumption tax and/or a tax on wage income the positive link between steady-state consumption growth rate γ_c and θ_i, $i = G, H$, weakens. The reason is the distorted consumption-leisure decision, i. e., the household tries to avoid the additional tax burden and substitutes leisure for labor. Hence, with endogenous labor supply, a consumption tax ceases to be lump-sum and impinges on the economy's growth rate.

Pareto Efficiency

The social planner chooses a plan (c, l, k) for each t to maximize household utility subject to the economy's resource constraint. Compared to the optimization of competitive households, the omniscient planner takes into account that the choice of l and k has an effect on the level of government consumption services. Since $H = \theta_H Y$ appears in the utility function this channel affects the constrained optimal steady-state growth rate of consumption.

Given l, the optimization generates the following expressions for the planner's choice of γ_c^P and γ_k^P:

$$\gamma_c^P = \frac{(1-\theta_G-\theta_H)}{1-b_h\Omega(l)}\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}} - \rho, \tag{3.46}$$

$$\gamma_k^P = (1-\theta_G-\theta_H)\frac{1-\Omega(l)(1+b_h)}{1-b_h\Omega(l)}\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}. \tag{3.47}$$

Here, $\Omega(l) \equiv (1/b_l)(\alpha/(1-\alpha)(l/1-l)$. The presence of $b_h\Omega(l)$ in the Euler equation (3.46) captures the fact that, for a given level of leisure/labor, the presence of utility-enhancing government consumption expenditure increases the benefits from capital investment today, lowering the consumption-output ratio and positively affecting the growth rate of consumption.[15] Further, the additional term $b_h\Omega(l)$ in the numerator of (3.47) reflects the

[15] The latter dominates the negative effect of leisure on the rate of return of private capital for any value of l if $b_h/b_l > 1$. If $0 < b_h/b_l < 1$, this only applies for not too small values of l.

fact that, from the planner's point of view, the marginal disutility of labor is lower since more labor means a higher consumption of H. Via this channel, the consumption-output ratio is lowered and the growth rate of capital is positively affected.

If we add the steady-state requirement $\gamma_c^P = \gamma_k^P$, then (3.46) and (3.47) give an expression for the constrained optimal steady-state growth rate that is similar to (3.36) with $\sigma_H = 1$ and a level of labor supply yet to be determined

$$\gamma_c^P = (1 - \theta_G - \theta_H) \left(AN^\alpha \theta_G^\alpha\right)^{\frac{1}{1-\alpha}} (1-l)^{\frac{\alpha}{1-\alpha}} - \frac{\rho}{1+b_h}. \tag{3.48}$$

In addition, (3.46) and (3.47) determine the steady-state labor supply implicitly.[16]

We can use our results to derive conditions under which a fiscal policy mix implements the constrained efficient allocation. This requires

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad (1-\tau_r)(1-\alpha) = \frac{1-\theta_G-\theta_H}{1-b_h\Omega(l)}, \tag{3.49}$$

$$\gamma_k = \gamma_k^P \quad \Leftrightarrow \quad \left(\frac{1-\tau_w}{1+\tau_c}\right)(1-\alpha)\Omega(l) = (1-\theta_G-\theta_H)\frac{\Omega(l)}{1-b_h\Omega(l)}. \tag{3.50}$$

According to the first of these conditions, τ_r has to be set such that the private marginal after-tax return on private capital equals the social rate of return on private capital. The second condition equalizes the consumption-output ratios of the equilibrium and planner's choice. It is then straightforward to see that the desired policy mix must satisfy the condition

$$(1-\tau_r)\left(\frac{1+\tau_c}{1-\tau_w}\right) = 1. \tag{3.51}$$

Intuitively, the effect of a distortionary tax on capital income can be offset by a compensating distortion of the consumption-leisure trade-off that strengthens labor supply. As long as lump-sum taxation is a feasible option any policy mix satisfying (3.51) is consistent with the government's budget constraint.[17]

If the social planner is allowed to pick θ_G and θ_H optimally, one finds that the optimal choice involves

$$\theta_G^P = \alpha \quad \text{and} \quad \theta_H^P = (1-\alpha)\, b_h\Omega(l), \tag{3.52}$$

i. e., the optimal share of productive government expenditure satisfies the natural condition of productive efficiency, and the optimal share of consumption expenditure is tied to the optimal level of leisure. Interestingly, from (3.49) and (3.50) the implementation of (θ_G^P, θ_H^P) is only possible if $\tau_r = 0$ and $\tau_w = -\tau_c$.

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[16] This condition is $\Omega(l)(1+b_h(1+z(l))) = z(l)$, where $z(l) \equiv \rho \left[(1-\theta_G-\theta_H)\left(A\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}\right]^{(-1)}$.

[17] Raurich (2003) studies optimal tax policies in the model of Turnovsky (2000a) assuming that neither lump-sum nor consumption taxes are admissible, yet the government's budget must be balanced in all periods.

3.6 Small Open Economy

Next, we turn to a small open economy with productive government expenditure, where agents are free to accumulate internationally traded bonds in a perfect world capital market. To highlight the role of openness we restrict attention to a pure public good such that the production function of household-producers is given by (3.2). Moreover, we abstract from the presence of public consumption services.

Since bonds and private capital are perfect substitutes as stores of value, in equilibrium they must pay the same after-tax rate of return, which is tied to the exogenous world interest rate $\bar{r}$. Hence, with government expenditure a fixed fraction of aggregate output according to (3.1) and an exogenous labor supply, this implies

$$(1-\tau_y)(1-\alpha)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}} = \bar{r}(1-\tau_b), \tag{3.53}$$

where τ_b is the tax rate on foreign bond income. Obviously, the pair of tax rates τ_y and τ_b that satisfies this condition cannot be chosen independently. To circumvent this problem, we introduce adjustment costs such that the price of installed capital, q, is variable and adjusts in equilibrium such that these after-tax rates of return are the same for any arbitrarily specified tax rates. The investment cost function is independent of government activity and given by

$$\varphi(i,k) \equiv \left(1+\frac{i}{2k}\right)i, \tag{3.54}$$

which simplifies (3.9) by fixing $\phi = 1$.

First, we study the case of an exogenous labor supply and then incorporate a labor-leisure trade-off. We shall see that the implications for government activity substantially differ in both cases. The exposition is based on Turnovsky (1999a).

3.6.1 Exogenous Labor Supply

Decentralized Equilibrium

Denote b the stock of net foreign bonds held by a household-producer at t and recall that k is the stock of capital in her (domestic) firm. Then, her flow budget constraint is given by

$$\dot{b} = \bar{r}(1-\tau_b)b + (1-\tau_y)y - (1+\tau_c)c - \varphi(i,k) - \tau. \tag{3.55}$$

The government budget modifies to $G = \tau_y Y + \tau_c C + \bar{r}\tau_b bN + \tau N$.

The objective of household-producers is to choose a plan (c, i, b, k) that maximizes utility (2.2) subject to (3.55) and $\dot{k} = i$. From the individual's optimality conditions with respect to c and b we obtain the Euler equation as

$$\gamma_c = \bar{r}(1-\tau_b) - \rho. \tag{3.56}$$

Hence, in a small open economy, the consumption growth rate is independent of domestic production conditions. It only depends on the given world interest rate, the tax rate on foreigns bonds, and on the rate of time preference.

The optimality conditions with respect to k and i deliver

$$q = 1 + \frac{i}{k}, \tag{3.57}$$

$$\bar{r}(1-\tau_b) = \frac{(1-\tau_y)(1-\alpha)\left(A\theta_G^\alpha N^\alpha\right)^{\frac{1}{1-\alpha}}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2q}. \tag{3.58}$$

As we saw in (3.12) the value of installed capital for an investing firm is greater than one. Equation (3.58) is a non-linear differential equation that describes the evolution of q such that the after-tax rate of return on traded bonds is equal to the after-tax rate of return on private domestic capital. The latter comprises the same elements as discussed following equation (3.14). Observe that (3.58) collapses to (3.53) for $q = 1$.

Turning to the steady state, we know from (3.57) that private domestic capital grows at the rate $\gamma_k = q - 1$, where q satisfies (3.58) for $\dot{q} = 0$.[18] Thus, the steady-state growth rate of capital (and output) depends on the domestic production technology as well as on various fiscal policy parameters. In contrast to the closed economy, consumption, capital and output generically grow at different rates, with the difference being reconciled by the accumulation of traded bonds.

As to the role of government activity on steady-state growth one finds that

$$\frac{d\gamma_k}{d\theta_G} > 0, \quad \frac{d\gamma_c}{d\theta_G} = 0; \quad \left.\frac{d\gamma_k}{d\theta_G}\right|_{\theta_G=\tau_y} \lesseqgtr 0 \quad \Leftrightarrow \quad \alpha \lesseqgtr \theta_G, \quad \left.\frac{d\gamma_c}{d\theta_G}\right|_{\theta_G=\tau_y} = 0. \tag{3.59}$$

The first two derivatives describe the effect of a rise in government expenditure financed through an adjustment in lump-sum taxes. The steady-state growth rate of capital increases since a higher θ_G increases the marginal product of capital such that the steady-state value of q in (3.58) increases; hence, $\gamma_k = q - 1$ rises. Due to (3.56), γ_c is independent of θ_G. The second two derivatives consider the benchmark case where government activity is only financed by income taxation, i. e., $\theta_G = \tau_y$. This introduces an offsetting effect on the steady-state value of q since a necessary rise in taxes reduces the after-tax marginal product of capital in (3.58). As in previous sections, there is a growth-maximizing share of government expenditure equal to α at which the price of installed capital is maximized.

The effect of τ_b on steady-state growth is given by

$$\frac{d\gamma_k}{d\tau_b} > 0, \quad \frac{d\gamma_c}{d\tau_b} < 0. \tag{3.60}$$

[18] The presence of convex adjustment costs may prevent the existence of a balanced growth path; for a discussion see Turnovsky (1996b). Further, the transversality condition of the household-producer's problem requires $\bar{r}(1-\tau_b) > \gamma_k$ and implies that only the smaller root of (3.58) is consistent with steady-state growth. Moreover, at this root, the right-hand side of (3.58) is negatively sloped.

Intuitively, an increase in the tax on bond income lowers the net rate of return on traded bonds, which requires a lower rate of return on installed capital, hence a higher q according to (3.58). Moreover, a higher τ_b reduces the households' willingness to postpone consumption and γ_c declines.

Pareto Efficiency

The planner maximizes u with respect to c, i, k, and b subject to $\dot{k} = i$ and the resource constraint

$$\dot{b} = (1 - \theta_G)y + \bar{r}b - c - \varphi(i, k)\,. \tag{3.61}$$

Accordingly, we obtain the constrained efficient steady-state growth rates of consumption and capital as

$$\gamma_c^P = \bar{r} - \rho \quad \text{and} \quad \gamma_k^P = q^P - 1, \tag{3.62}$$

where q^P is determined by

$$\bar{r} = \frac{(1-\theta_G)\left(A\theta_G^{\alpha}N^{\alpha}\right)^{\frac{1}{1-\alpha}}}{q^P} + \frac{(q^P - 1)^2}{2q^P}. \tag{3.63}$$

The interpretation of (3.62) and (3.63) mimics the one of (3.56) and (3.58) in the decentralized equilibrium. Due to the presence of τ_b, we have $\gamma_c^P > \gamma_c$. It follows that $\tau_b = 0$ is necessary to implement the constrained efficient allocation. Then, for the same reasons set out in Footnote 18, we find

$$q \lesseqqgtr q^P \quad \Leftrightarrow \quad \gamma_k \lesseqqgtr \gamma_k^P \quad \Leftrightarrow \quad \frac{\theta_G - \alpha}{1-\alpha} \lesseqqgtr \tau_y. \tag{3.64}$$

Allowing the social planner to additionally determine the optimal size of the government reveals that the growth-maximizing share of government expenditure is also welfare-maximizing, i.e. $\theta_G^P = \alpha$. If $\theta_G \neq \alpha$, we obtain that capital and interest income should be taxed at different rates. This result is driven by the assumption that government expenditure is a fixed fraction of output and thereby independent of interest income.

3.6.2 Endogenous Labor Supply

In this section we introduce an endogenous labor supply in the small open economy of the previous section.

Decentralized Equilibrium

The household-producer chooses a plan (c, l, i, b, k) to maximize

$$u \;=\; \int_0^\infty \left[\ln c + b_l \,\ln l\right] e^{-\rho t}\, dt \tag{3.65}$$

subject to $\dot{k} = i$, the budget constraint $\dot{b} = (1-\tau_w)w(1-l) + (1-\tau_r)rk + \bar{r}(1-\tau_b)b - (1+\tau_c)c - \varphi(i,k) - \tau$, and the production function (3.43). This leads to the conditions for consumption and domestic capital growth (3.56) and (3.57) as well as the following optimality conditions

$$\bar{r}(1-\tau_b) = \frac{(1-\tau_r)(1-\alpha)\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}}{q} + \frac{\dot{q}}{q} + \frac{(q-1)^2}{2q}, \tag{3.66}$$

$$\frac{c}{y} = \left(\frac{1-\tau_w}{1+\tau_c}\right)\frac{\alpha}{b_l}\left(\frac{l}{1-l}\right). \tag{3.67}$$

In the steady state $\dot{q} = 0$ such that (3.66) determines the equilibrium price of installed capital given l. Condition (3.67) implies that in a steady state with constant labor supply c and y must grow at the same rate. Moreover, one can show that in a steady state y, k, and G must grow at the same rate. It follows that

$$\gamma_k = q - 1 = \bar{r}(1-\tau_b) - \rho = \gamma_c. \tag{3.68}$$

Equation (3.68) implies that, contrary to the case with exogenous labor supply, in equilibrium capital, output, and consumption grow at the same rate determined by the net interest rate on foreign bonds and the rate of time preference. Hence, with endogenous labor supply the production side is irrelevant for the steady-state growth rates of consumption *and* domestic capital.

From (3.68) it follows that out of the set of fiscal policy variables, only changes in τ_b generate steady-state growth effects. The reason is that (3.68) also pins down q. Therefore, in a steady state changes in θ_G, τ_k, and τ_b lead to adjustments of labor supply such that (3.66) remains valid. One readily verifies that $dl/d\theta_G > 0$, $dl/d\tau_k < 0$, and $dl/d\tau_b > 0$. Moreover, since τ_w and τ_c do not show up in (3.66) it follows that these taxes are essentially lump-sum, i. e., $dl/d\tau_w = dl/d\tau_c = 0$. This is in stark contrast to the results obtained under endogenous labor supply in the closed economy of Section 3.5.

Pareto Efficiency

The social planner chooses a plan (c, l, i, b, k) to maximize individual utility (3.65) subject to $\dot{k} = i$ and the resource constraint (3.61) where y is given by (3.43). Following the same procedure as in the decentralized setting we obtain the steady-state conditions

$$\gamma_k^P = q^P - 1 = \bar{r} - \rho = \gamma_c^P \tag{3.69}$$

$$\bar{r} = \frac{(1-\theta_G)\left(AN^\alpha\theta_G^\alpha\right)^{\frac{1}{1-\alpha}}(1-l)^{\frac{\alpha}{1-\alpha}}}{q^P} + \frac{(q^P-1)^2}{2q^P} \tag{3.70}$$

$$\frac{c}{y} = \left(\frac{1-\theta_G}{1-\alpha}\right)\frac{\alpha}{b_l}\left(\frac{l}{1-l}\right). \tag{3.71}$$

The tax rates that replicate the constrained efficient steady-state path, bring (3.66) - (3.68) in line with (3.69) - (3.71). These are $\tau_b = 0$ and $(1-\tau_r)(1+\tau_c)/(1-\tau_w) = 1$, where the latter is a restatement of condition (3.51) derived for the closed economy. The welfare-maximizing share of government expenditure is equal to α. Moreover, with $\theta_G = \theta_G^P$, the optimal tax rates can be shown to be $\tau_r = 0$ and $\tau_w = -\tau_c$. This confirms the results of the small open economy with exogenous labor supply and for the closed economy with endogenous labor supply. However, here the choice of θ_G^P does not have a growth effect but assures the static efficiency of the steady state.

4 Productive Government Activity as a Stock

The difference between the stock and the flow approach to modeling productive government activity is that $G(t)$ is not provided out of current output but results from past public investments, i. e., $G(t)$ is the aggregate stock of public capital at t.

The first paper that treats productive government activity as a stock in our analytical framework is Futagami, Morita, and Shibata (1993). These authors assume that the public capital stock is a pure public good such that $g = G$. Here, we begin our discussion of the stock approach by directly allowing for the congestion of public services. Then, we incorporate two aspects that arise only if we think of productive government activity as a stock.[19]

4.1 Public Goods Subject to Congestion

We follow Turnovsky (1997a) and assume that current public investment is a constant fraction of aggregate output denoted by $\theta_G \in (0, 1)$. We abstract from depreciation such that G evolves according to

$$\dot{G} = \theta_G Y. \tag{4.1}$$

The household-producer's production technology continues to be as in (2.1). As a consequence, in the stock case G will only be a constant fraction of Y in the steady state, whereas in the flow case this holds for all t in accordance with condition (3.1).

[19] The stock modeling approach has incorporated many facets that we will not discuss in detail. For instance, Lau (1995) and Chen (2006) incorporate public consumption expenditure affecting the per-period utility function. See Baier and Glomm (2001) and Raurich-Puigdevall (2000) for stock models with an endogenous labor supply. Turnovsky (1997b) is the reference for a small open economy. This framework is applied by Chatterjee, Sakoulis, and Turnovsky (2003) to analyze the process of developmental assistance through unilateral capital transfers tied to investment in public capital. Gómez (2004) devises a fiscal policy that allows to implement the Pareto-efficient allocation when investments are irreversible. Devarajan, Xie, and Zou (1998) study alternative ways how to provide public capital.

Let the service derived by the individual household-producer g be given by (3.18). As in the flow model of Section 3.3.1, the individual household-producer chooses c and k to maximize utility u of (2.2) subject to her flow budget constraint (2.3) and the production function (3.19) which we repeat here for convenience

$$y = A\left(\frac{G}{K}\right)^{\alpha}\left(\frac{k}{K}\right)^{-\sigma_G\,\alpha} k.$$

In her intertemporal optimization the individual household-producer neglects her impact on the aggregate private capital stock K and takes the stock of public capital G as given. Then, the Euler condition obtains as

$$\gamma_c = (1-\tau_y)(1-\sigma_G\alpha)AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} - \rho \equiv \gamma_c\left(\frac{G}{k}, \tau_y\right), \tag{4.2}$$

where we use the fact that in equilibrium $K = Nk$.

This growth rate looks similar to the Euler condition in the flow model (see, e. g., equation (3.3) where $\sigma_G = 1$). Again, the first term on the right-hand side of the Euler equation is the private marginal product of private capital. In the flow model, the ratio G/k is determined by exogenous parameters since G is proportionate to Y at all t. Therefore, the growth rate of consumption is time-invariant. Here, this is not the case since the proportionality of G and Y occurs only in the steady state. As a consequence, additional differential equations are needed to fully characterize the dynamical system.

To derive these conditions, we divide the aggregate resource constraint (3.6) by k and the public accumulation equation (4.1) by G. Taking into account that equilibrium production is given by

$$y = AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} k, \tag{4.3}$$

we find two additional differential equations in G and k

$$\gamma_G = \theta_G AN^{\alpha(\sigma_G-1)+1}\left(\frac{G}{k}\right)^{\alpha-1} \equiv \gamma_G\left(\frac{G}{k}\right), \tag{4.4}$$

$$\gamma_k = (1-\theta_G)\,AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} - \frac{c}{k}. \tag{4.5}$$

The dynamical system of the economy is then described by equations (4.2), (4.4), and (4.5) in conjunction with initial conditions k_0, G_0, and the transversality condition of the household-producer's optimization problem.

Here, we focus on the steady state and its properties. From (4.2) and (4.3), G, k, and y have the same steady-state growth rate. This growth rate and the steady-state ratio, $(G/k)|_{ss}$, can be obtained from (4.2) and (4.4). Figure 2 illustrates the loci γ_c and γ_G as functions of G/k.

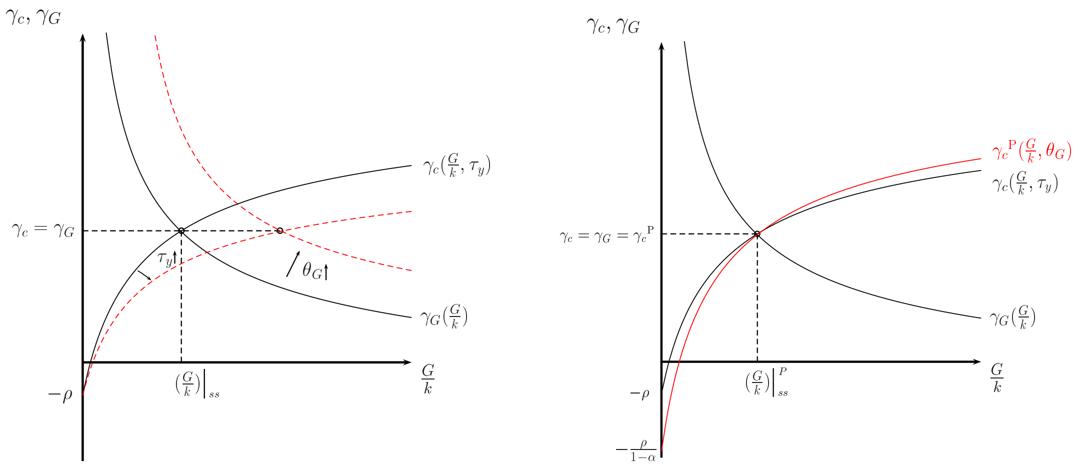

Figure 2: Steady-State Growth Rates: Decentralized Equilibrium (left) and the Implementation of the Pareto-Efficient allocation (right).

Next, we turn to the effect of fiscal policy variables on steady-state growth. A lump-sum financed increase in the share of government investment, θ_G, corresponds to an upward shift of the γ_G-locus in Figure 2, which implies a higher steady-state growth rate. If instead of a lump-sum tax a distortionary income tax is used for funding such that $\theta_G = \tau_y$, then in addition the γ_c-locus pivots downwards. The overall effect on the steady-state growth rate depends on the relative strength of both shifts. Analytically, one can show that

$$\frac{d\gamma_c}{d\theta_G} \gtreqless 0 \quad \Leftrightarrow \quad \alpha \gtreqless \theta_G. \tag{4.6}$$

Hence, as in the flow model, the growth-maximizing share of government investment is $\theta_G^* = \tau_y^* = \alpha$.

Pareto Efficiency

In contrast to the individual household-producers the social planner not only chooses c and k but also the public capital stock G to maximize utility (2.2) subject to the aggregate resource constraint (3.6) and the accumulation equation of public capital (4.1) with aggregate production y given by (4.3). This problem delivers the steady-state Euler equation[20]

$$\gamma_c^P = (1 - \theta_G)\frac{\partial y}{\partial k} + \theta_G N \frac{\partial y}{\partial G} - \rho. \tag{4.7}$$

The first two terms on the right-hand side have an interpretation as the social return of an additional marginal unit of output. Along the optimal path, the planner allocates the fraction $1 - \theta_G$ of this unit to private capital and the fraction θ_G to public capital. This partition is imposed by the public accumulation equation (4.1). The second term

[20] A detailed derivation of this and other results discussed in this section can be found in the Appendix.

corresponds to the benefit of a marginal increase in the provision of public capital associated with θ_G units of current output. Since the planner views G as a pure public good, the marginal increase in aggregate output is N times the marginal increase in individual output.

In light of (4.3) and (4.4), (4.7) can be written as

$$\gamma_c^P = (1-\theta_G)(1-\alpha) AN^{\alpha(\sigma_G - 1)} \left(\frac{G}{k}\right)^{\alpha} + \alpha\gamma_G - \rho. \tag{4.8}$$

In a steady state $\gamma_c^P = \gamma_G$ such that (4.8) becomes

$$\gamma_c^P = (1-\theta_G) AN^{\alpha(\sigma_G - 1)} \left(\frac{G}{k}\right)^{\alpha} - \frac{\rho}{1-\alpha} \equiv \gamma_c^P\left(\frac{G}{k}, \theta_G\right). \tag{4.9}$$

The steady-state ratio $(G/k)|_{ss}^P$ is then determined by the conditions (4.9) and (4.4). See Figure 2 for an illustration.

Comparing γ_c of (4.2) to γ_c^P of (4.9) shows that

$$\gamma_c = \gamma_c^P \quad \Leftrightarrow \quad AN^{\alpha(\sigma_G - 1)} \left(\frac{G}{k}\right)^{\alpha} [(1-\tau_y)(1-\sigma_G\,\alpha) - (1-\theta_G)] = \frac{-\rho\alpha}{1-\alpha}. \tag{4.10}$$

Hence, an income tax rate τ_y that implements $(G/k)|_{ss}^P$ given θ_G exists. It is lower the lower the degree of congestion (i. e., the larger σ_G) and higher the greater θ_G.

Allowing the planner to choose θ_G optimally delivers

$$\theta_G^P = \alpha - \frac{\rho}{AN^{\sigma_G\,\alpha}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} < \alpha. \tag{4.11}$$

Interestingly, θ_G^P falls short of the growth-maximizing level $\theta_G^* = \alpha$. This difference occurs as the advantage of a larger public investment share materializes only tomorrow whereas the cost in terms of foregone consumption is to be paid today. This intertemporal aspect explains why θ_G^P declines in ρ. Since the benefit of an increase in the stock of public capital accrues to all firms, θ_G^P increases in N. Notice, that no intertemporal consideration is present when θ_G^P is determined in the flow model of Section 3.3.1. Therefore, in that case the growth-maximizing and the welfare-maximizing expenditure shares coincide, i. e., $\theta_G^* = \theta_G^P = \alpha$.

One can show that an income tax rate equal to

$$\tau_y^P = \frac{\alpha(1-\sigma_G)}{1-\sigma_G\,\alpha}, \tag{4.12}$$

implements the Pareto-efficient steady-state allocation involving θ_G^P. The optimal income tax corrects for the congestion externality and recommends the same tax rate as in the flow model of Section 3.3.1 (see equation (3.23)). The larger the degree of congestion the greater the optimal income tax.[21]

A curious implication arises when the degree of congestion is sufficiently high, i. e., σ_G close to zero. For instance, in the extreme case of proportional congestion, $\sigma_G = 0$, $\tau_y^P = \alpha > \theta_G^P$ such that the government should impose an income tax rate in excess of its current investment costs and refund the excess revenue in form of lump-sum taxes. In the respective flow model (equation (3.23)) the optimal income tax is $\tau_y^P = \alpha = \theta_G^P$ so that government expenditure is exactly covered. Thus, in the stock model a larger income tax rate is required in order to offset the incentive to overaccumulate private capital due to congestion.

4.2 Maintenance of Public Capital

Due to its use or the passage of time, a fraction of the current stock of public capital depreciates. Maintenance refers to investments that replace depreciated public capital. Conceptually, the incorporation of such replacement investments requires the identification of wear and tear with different parts of the existing public capital stock. Since here this stock comprises homogeneous capital goods, such an identification is not possible. Therefore, we follow the literature, in particular Rioja (2003) and Kalaitzidakis and Kalyvitis (2004), and model replacement investments as an attempt to reduce the instantaneous rate of depreciation of public capital. The new question is then how the economy splits up its expenditure on public capital into "new" public capital goods and in replacement investments, i. e., investments that reduce the rate of depreciation.

Denote G_I the per-period investments in "new" public capital goods and M the level of per-period maintenance investments. Then, the economy's gross investment is $G_I + M$. With D denoting depreciation, the stock of public capital evolves according to $\dot{G} = G_I + M - D$.

As proposed by Kalaitzidakis and Kalyvitis (2004), we model the difference between replacement investments and actual depreciation as

$$M - D \equiv \delta_G\left(\frac{M}{Y}\right)G, \quad \text{with} \quad \delta_G(.) > 0 > \delta_G'(.). \tag{4.13}$$

The idea is that a higher level of maintenance M reduces the level of depreciation whereas a more intense usage measured by Y increases it. With (4.13) the accumulation of public

[21] Marrero and Novales (2005) show that the presence of a significant level of wasteful public expenditure that does neither affect the economy's technology nor preferences is another reason for why a positive income tax leads to faster long-run growth and higher welfare than lump-sum taxes. Turnovsky (1997b) confirms the results of (4.11) and (4.12) for a small open economy with exogenous labor supply and private and public investments subject to adjustment costs.

capital is governed by

$$\dot{G} = G_I - \delta_G\left(\frac{M}{Y}\right)G. \tag{4.14}$$

We assume that the government finances its total expenditure, $G_I + M$, via income taxes such that the government's budget constraint is

$$M + G_I = \tau_y Y. \tag{4.15}$$

Let θ_M and $(1-\theta_M)$ denote the shares of total government expenditure that are allocated to maintenance and "new" capital goods, respectively, i. e.,

$$M = \theta_M \tau_y Y \quad \text{and} \quad G_I = (1-\theta_M)\tau_y Y. \tag{4.16}$$

To simplify, we abstract from congestion effects such that the individual household-producer's production function is (3.2) which we restate here for convenience

$$y = A\left(\frac{G}{k}\right)^{\alpha} k.$$

Decentralized Equilibrium

The individual household-producer chooses c and k to maximize her utility u given by (2.2) subject to her flow budget constraint $\dot{k} = (1-\tau_y)y - c$. The Euler condition is then

$$\gamma_c = (1-\tau_y)(1-\alpha)A\left(\frac{G}{k}\right)^{\alpha} - \rho, \tag{4.17}$$

which corresponds to (4.2) for $\sigma_G = 1$. The growth rates of public and private capital result from the public accumulation equation (4.14) and the individual's resource constraint

$$\gamma_G = (1-\theta_M)\tau_y AN\left(\frac{G}{k}\right)^{\alpha-1} - \delta_G(\theta_M\tau_y), \tag{4.18}$$

$$\gamma_k = (1-\tau_y)\,A\left(\frac{G}{k}\right)^{\alpha} - \frac{c}{k}. \tag{4.19}$$

Then, the dynamical system of the economy is given by (4.17)-(4.19) and initial conditions k_0, G_0, and the transversality condition of the household-producer's optimization problem.

Analogously to the previous section, we obtain the steady-state ratio $(G/k)|_{ss}$ and the common steady-state growth rate for c, G, and k from (4.17) and (4.18). These equations also reveal that no clear cut comparative statics for the steady-state growth rate with respect to θ_M and τ_y are available. However, a steady-state growth-maximizing share of maintenance investments, θ_M^*, can be determined, at least implicitly. The total differential of (4.17) and (4.18) delivers the condition

$$AN\left[\left(\frac{G}{k}\right)^{*}\right]^{\alpha-1} = -\delta_G'\left(\theta_M^*\tau_y\right). \tag{4.20}$$

Intuitively, the optimal allocation of current output to public capital investments satisfies $\partial\dot{G}/\partial G_I = \partial\dot{G}/\partial M$, i.e., the last marginal unit spent on maintenance contributes the same amount to the change in public capital stock as the last marginal unit spent on "new" public capital goods.

Kalaitzidakis and Kalyvitis (2004) show further that the growth-maximizing income tax rate, τ_y^*, evaluated at $\theta_M = \theta_M^*$ is

$$\tau_y^* = \frac{\alpha}{1-\theta_M^*(1-\alpha)} > \alpha. \tag{4.21}$$

This result contrasts with the finding of the previous sections where the growth-maximizing tax rate was found to equal α. Intuitively, the presence of maintenance adds a productive use to public capital expenditure. To exploit this opportunity, the optimal income tax should be higher than without it. To strengthen this intuition we introduce an explicit functional form such that $\delta_G = (\theta_M \tau_y)^{-\varepsilon}$, $\varepsilon > 0$. Then, τ_y^* of (4.21) becomes

$$\tau_y^* = \frac{\alpha\,(\varepsilon+1)}{1+\alpha\varepsilon}. \tag{4.22}$$

In the limit $\varepsilon \to 0$, the effect of maintenance vanishes and the optimal income tax is $\tau_y^* = \alpha$. On the other hand, the effect of maintenance becomes more pronounced the larger ε and $\tau_y^* \to 1$ as $\varepsilon \to \infty$.[22]

Further, it can be shown that the growth-maximizing share of new public capital goods, $(G_I/Y)^* = (1-\theta_M^*)\tau_y^* < \alpha$. With $\delta_G = (\theta_M\tau_y)^{-\varepsilon}$, we find using (4.22) in (4.21) that $\theta_M^* = \varepsilon/\,(\varepsilon+1)$. Then,

$$(G_I/Y)^* = \frac{\alpha}{1+\alpha\varepsilon} < \alpha. \tag{4.23}$$

Hence, for $\varepsilon = 0$ we are back in the case without maintenance and $(G_I/Y)^* = \alpha$. Moreover, as $\varepsilon \to \infty$ all public expenditure goes to maintenance and $(G_I/Y)^* \to 0$.

4.3 Stock-Flow Model of Public Goods

Thus far, we have considered either the flow or the stock approach to modeling public services. An interesting question taken up by Tsoukis and Miller (2003) and Ghosh and

[22]Similar to the present setup, Greiner and Hanusch (1998) have a stock model where government expenditure can be allocated to two uses. They are the accumulation of the public capital stock and a subsidy to private capital accumulation. The point of their paper is that a rise in the subsidy rate for private capital investment is not necessarily growth-enhancing because it diverts resources away from productive government spending. Moreover, these authors show that for strictly positive subsidy rates the growth-maximizing income tax rate is strictly greater than α. Hence, the qualitative finding of (4.21) may also be the consequence of a growth policy that strengthens the investment incentives of private firms.

Roy (2004) is whether and how new implications for growth and welfare arise if both approaches appear simultaneously.

Let G_f denote the flow of public services and G_s the stock of public capital. Then, a natural extension of the production function (3.2) is

$$y = \left(G_s^{\beta} G_f^{1-\beta}\right)^{\alpha} k^{1-\alpha}, \quad 0 < \beta < 1. \tag{4.24}$$

We assume that $\dot{G}_s = \theta_{G_s} Y$ and $G_f = \theta_{G_f} Y$. Moreover, total government expenditure is fully financed via a distortionary income tax and continues to be a fixed fraction of output, i. e.,

$$\dot{G}_s + G_f = \tau_y Y = \theta_G Y, \quad \theta_G \equiv \theta_{G_s} + \theta_{G_f}. \tag{4.25}$$

Tsoukis and Miller (2003) show that the growth-maximizing shares are

$$\theta_G^* = \alpha, \quad \theta_{G_s}^* = \alpha\beta, \qquad \theta_{G_f}^* = \alpha(1-\beta). \tag{4.26}$$

Hence, each facet of public expenditure receives a share equal to its respective output elasticity.

The Pareto-efficient allocation mimics the properties of the previous sections. In particular, one finds that the equilibrium shares of total expenditure and of public capital investment are too large relative to their welfare-maximizing level whereas the equilibrium flow share is the welfare-maximizing one, i. e.,

$$\theta_G^P < \theta_G^*, \quad \theta_{G_s}^P < \theta_{G_s}^*, \quad \theta_{G_f}^P = \theta_{G_f}^*. \tag{4.27}$$

Ghosh and Roy (2004) analyze the question how the government by deciding on the ratio of the two types of public spending can at least partially compensate for the non-optimal choices of the private sector.

5 Variations on a Theme

5.1 Stochastic Environments

Turnovsky (1999c) studies the role of productive government expenditure in a stochastic version of the flow model with congestion as presented in Section 3.3.1. He finds that under uncertainty the growth-maximizing level of government expenditure depends on the degree of relative risk aversion. If the latter is strong, then the growth-maximizing expenditure share exceeds the Pareto-efficient one.

On the production side, uncertainty is introduced via a productivity shock, du, that is i.i.d. - normal with zero mean and variance $\sigma_u^2 dt > 0$. This shock is proportional to the current mean flow of output. More precisely, the flow of output, dy, produced by the individual household-producer over the small time period $(t, t + dt)$ is $dy = Ag^{\alpha}k^{1-\alpha}[dt + du]$, where g is given by (3.18). Government expenditure comprises a deterministic, productivity-enhancing component, G, and a stochastic component, G'. The total flow of resources claimed by the government over the period dt amounts to $d\bar{G} = Gdt + G'du$. Both types of government expenditure are fixed fractions of the aggregate mean rate of the output flow, i. e. $G = \theta_G NAg^{\alpha}k^{1-\alpha}$ and $G' = \theta_G^{'} NAg^{\alpha}k^{1-\alpha}$. Thus, the fraction θ_G now represents the government's choice of the (deterministic) size of government, while $\theta_G^{'}$ represents the fraction of the aggregate output shock absorbed by the government.

To allow for varying degrees of risk aversion the per-period utility function is now $\left(c^{1-v} - 1\right)/(1 - v)$, $v \geq 1$. Here, v is the coefficient of relative risk aversion.

This setting delivers a unique stochastic balanced growth path where the mean growth rate depends on the degree of relative risk aversion, the variance of the shock, the shares of government expenditure, and the degree of relative congestion. With $\sigma_u^2 = 0$ and $v = 1$ this growth rate collapses to the one under certainty as given by (3.20). To interpret the equilibrium under uncertainty we follow Turnovsky (1999c) and consider reasonable degrees of relative risk aversion to be $v > 1$.

The mean steady-state growth rate increases in the variance of du. Intuitively, a higher variance of the shocks means higher risk. Therefore, more risk-averse agents increase their precautionary savings, which allows for faster growth.

The deterministic growth-maximizing share of government expenditure under full income tax financing, θ_G^*, exceeds α. The reason is that a higher θ_G raises the productivity of private capital and, since the shock is proportional to output, magnifies the volatility of output. As the latter induces more precautionary savings that increase the mean growth rate there is an additional reason to increase θ_G.

The introduction of uncertainty reduces the Pareto-efficient share of deterministic government expenditure below θ_G^*. Intuitively, the planner takes the individual's risk aversion into account and chooses a smaller steady-state growth rate that comes along with lower volatility. The optimal tax structure that implements the Pareto-efficient allocation has to internalize the congestion externality. This is accomplished with a strictly positive income tax. This tax reduces the growth rate of the economy and, hence, the degree of volatility.[23]

[23]Turnovsky (1999b) considers a small open economy under the same uncertainty as above. He shows that the Pareto-efficient share of government expenditure is greater in the open than in the closed economy

5.2 Increasing Returns

Thus far, we have assumed that the production function of the individual firm exhibits constant returns to scale with respect to private capital and productive government expenditure at the social level. Constant returns are, among others, responsible for the existence of a balanced growth path and the absence of transitional dynamics in the flow models based on Barro (1990). Intuitively, this assumption is not mandatory. For instance, in developing countries the density of the road network may be so low that twice as much private capital and twice as many roads more than double output.

Conceptually, in the presence of external effects associated with productive government expenditure, the expected return on private capital investments of individual firms depends on the investment decisions of all other firms. Thus, there is scope for a self-fulfilling prophecy (Krugman (1991)). If all household-producers believe the return on investment to be high, they will invest a lot today. Then, tomorrow aggregate output and, accordingly, government expenditure will be large. The latter raises the return on investment such that the belief of a high rate of return is confirmed in equilibrium.

Abe (1995) and Zhang (2000) incorporate increasing returns at the social level into the flow setup and find multiple equilibria and sophisticated transitional dynamics.[24] For instance, the dynamical system of Abe (1995) delivers a new locally-stable and stationary steady state in addition to an endogenous growth path. Accordingly, the economy may be trapped in a sufficiently small neighborhood of the stationary steady state. Alternatively, a coordinated hike in investment activity may push the economy sufficiently far away from this steady state such that it embarks on an endogenous growth path. The latter may be either due to a self-fulfilling prophecy or to an unpredicted and temporary rise in government activity.[25]

5.3 Non-Scale Growth

In previous sections, we have emphasized that the steady-state growth rate depends on the size of the economy measured by the "number" of household-producers - at least as

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if and only if the economy is a net creditor. The reason is that some of the risk of domestic productivity shocks is exported and reduces the volatility of domestic income. Hence, for a given degree of risk aversion, the individual is ready to accept a greater volatility caused by a bigger size of the government.

[24]Both authors generalize the production function (3.2) to $y = AG^{\alpha}k^{\beta}$ where $\alpha + \beta > 1$. Moreover, they allow for the public good to affect per-period utility. Abe (1995) adopts the research production function of Romer (1986), p. 1019, to model capital accumulation.

[25]Some details necessary to guarantee the success of the suggested government intervention are quite involved. Zhang (2000) reaches similar policy conclusions, e. g., when his interior stationary steady state is an unstable focus.

long as the provision of the public good has an element of non-rivalry (see footnote 5). The larger N, the faster the economy grows. This finding is often referred to as the *scale effect* and has been criticized on both empirical and theoretical grounds (Jones (1995)).[26] Here, it arises since the level of government expenditure is tied to the size of the economy measured by aggregate output Ny.

Eicher and Turnovsky (2000) study productive government spending as a flow in a non-scale endogenous growth model in the spirit of Jones (1995). As new elements, their approach incorporates population growth, i. e., a constant growth rate of the "number" of household-producers, $\gamma_N \neq 0$, and a simultaneous treatment of relative and aggregate congestion of public services.[27] The latter is achieved with a modification of equation (3.18). Here, the functional form of productive services derived by an individual firm from public expenditure is

$$g = G \left(\frac{k}{K} \right)^{1-\sigma_R} K^{\sigma_A - 1}, \tag{5.1}$$

where $\sigma_R, \sigma_A \in [0, 1]$ parameterize the degree of relative and aggregate congestion, respectively. Clearly, $\sigma_R = \sigma_A = 1$ is the special case of a pure public good.

In addition, Eicher and Turnovsky (2000) allow for increasing or decreasing returns to scale in the production function of the individual firm such that (2.1) is replaced by $y = Ak^{\beta} g^{\alpha}$ with $\alpha, \beta \in [0, 1]$. Upon combining this production function, (5.1), and (3.1) one obtains at the social level

$$y = (A\theta_G^{\alpha})^{\frac{1}{1-\alpha}} \, k^{\frac{\alpha(\sigma_A - 1) + \beta}{1-\alpha}} N^{\frac{\alpha(\sigma_R + \sigma_A - 1)}{1-\alpha}}. \tag{5.2}$$

The latter is consistent with a balanced growth path involving $\gamma_c = \gamma_k = \gamma_y$ if

$$\gamma_c = \frac{\alpha\,(\sigma_R + \sigma_A - 1)}{1 - \beta - \alpha\sigma_A} \gamma_N \gtreqless 0. \tag{5.3}$$

To fix ideas, assume that the marginal product of capital in (5.2) is strictly positive, i. e., $\alpha(\sigma_A - 1) + \beta > 0$, and let $\gamma_N > 0$. If the denominator of (5.3) is positive, then the marginal product of capital is decreasing in (5.2). As a consequence, y cannot grow as fast as k unless some of the growth of y is due to population growth. Indeed, the numerator is only strictly positive if the output elasticity of labor is positive such that population

[26] The scale effect is a feature of the first-generation endogenous growth models of Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992). Here, it results from the specification of the production function for new knowledge. Subsequent idea-based growth models follow Jones (1995) and modify this functional relationship to find qualitatively similar steady-state growth rates as the one derived in equation (5.3). See Jones (1999) for a concise summary of this literature and Eicher and Turnovsky (1999) for a general treatment.

[27] Pintea and Turnovsky (2006) study the role of relative and aggregate congestion in a two-sector non-scale model with private and "public" firms.

growth contributes positively to the growth of y. In turn, this is the case if the degrees of congestion are not too pronounced.

The way we find the steady-state growth rate of (5.3) is quite different from previous sections. In fact, here we are not concerned with first-order conditions to determine intertemporal prices and, hence, the households' Euler condition. Instead, we require the consistency of equal growth rates of per-capita magnitudes with the economy's technology given by (5.2). As a result, the steady-state growth rate is independent of preference parameters like ρ or fiscal policy variables such as θ_G. Consequently, the derivation of a growth-maximizing share of government expenditure θ_G^* as discussed in Section 3 becomes irrelevant.

By contrast, a welfare-maximizing share of government expenditure, θ_G^P, can still be determined since the static allocation consistent with steady-state growth need not be efficient. Eicher and Turnovsky (2000) show that $\theta_G^P = \alpha$. Moreover, there is a time-invariant income tax rate that implements the Pareto-efficient allocation $\tau_y^P = \alpha(2 - \sigma_R - \sigma_A)/(\beta + \alpha(1 - \sigma_R))$. Intuitively, τ_y^P internalizes both externalities caused by relative and aggregate congestion. Clearly, τ_y^P decreases in σ_R and σ_A.

6 Concluding Remarks

What is the role of productive government expenditure for sustained economic growth? The literature surveyed in this paper provides a rich set of hints to a full-fledged answer.

First, it establishes an analytical framework in which productive government activity is necessary for balanced growth of per-capita magnitudes. Without government activity, we would be back in the neoclassical growth model without technical change and sustained long-run growth. In this framework, government activity can be treated either as a flow or as a stock. In both cases the technology of the economy has the following properties. At the level of individual firms, there are constant returns to scale with respect to private capital, k, and the services derived from productive government activity, g. At the social level, two assumptions imply that the production function of the individual firm becomes, at least asymptotically, linear in k. First, services, g, derived by individual firms are proportional to the level of total government activity, G. Second, the current flow of government expenditure is proportional to the size of the economy. In the flow case, since $G = \theta_G Y$ the linearity in k holds at all t; in the stock case, since $\dot{G} = \theta_G Y$, this linearity holds only in the steady state.

As a consequence, the steady-state properties of the scenarios under scrutiny are similar to those of the AK-model such that the Euler equation determines the steady-state growth rate. We use this property to study and compare the link between productive government activity, economic growth, and welfare in different economic settings.

Second, productive government expenditure impacts on the steady-state growth rate of consumption through a direct effect on the technology and an indirect effect on investment incentives through the mode of financing. The direct effect is strictly positive except for the small open economy where consumption growth is determined by parameters that are exogenous to the domestic economy. This can be verified from the first column of Table I. It shows the effect of a larger government share, θ_G, on consumption growth under full lump-sum financing. Another polar case has full income tax financing. Such a tax reduces the after-tax marginal return on private capital. Hence, the indirect effect on consumption growth is strictly negative. Column 2 in Table I reveals that these opposing forces tend to give rise to a growth-maximizing government share. In most settings, this share is equal to the output elasticity of the public input, α. If the government service in addition reduces adjustment costs, then $\theta_G^* > \alpha$; if the government also provides consumption services, then $\theta_G^* < \alpha$.

Table I: Summary of the Main Findings

	$\frac{d\gamma_c}{d\theta_G}$	θ_G^*	θ_G^P	τ_y^P
Flow Modeling				
1. Pure Public Good Case	+	α	α	0
2. Adjustment Costs	+	$> \alpha$	$> \alpha$	> 0
3.1 Public Goods Subject to Congestion Without Excludability	+	α	α	> 0 (for $\sigma_G \neq 1$)
3.2 Public Goods Subject to Congestion With Excludability	+	α	α	$\begin{cases} 0 & \sigma_G = \sigma_E = 1 \\ > 0 & otherwise \end{cases}$
4. Public Consumption Services	+	$\alpha(1-\theta_H)$	$0 < \theta_G^P \leq \alpha$	$(1-\sigma_H)\theta_H^P$
5. Endogenous Labor Supply	+	N.A.	α	$\tau_r = 0,\ \tau_w = -\tau_c$
6.1 SOE: Exogenous Labor Supply	0	N.A.	α	$\tau_y^P = \tau_b^P = 0$
6.2 SOE: Endogenous Labor Supply	0	N.A.	α	$\tau_r = 0,\ \tau_w = -\tau_c$
Stock Modeling				
Relative Congestion Without Excludability	+	α	$< \alpha$	$\begin{cases} > 0 & \sigma_G \in [0,1) \\ = 0 & \sigma_G = 1 \end{cases}$

Third, the welfare-maximizing, i. e., Pareto-efficient, share of government expenditure - Column 3 of Table I - need not coincide with the growth-maximizing government share. This reflects the trade-off involved in the consumption-savings decision that the planner takes into account: faster growth requires higher investment outlays and reduces consumption today. Most interestingly, here the difference between the flow and the stock variant matters. In the stock case, the benefit from government expenditure today is smaller since it augments output only tomorrow. Therefore, the welfare-maximizing share of government expenditure is smaller.

Fourth, as shown in Column 4 of Table I, appropriate fiscal policy measures can implement the Pareto-efficient allocation. Intuitively, a strictly positive income tax can be used to correct for overaccumulation of private capital due to a negative externality such as congestion.

Arguably, within this well-defined analytical framework further facets of the link between productive government expenditure and sustained economic growth can be studied. One important aspect for economic growth is the government's ability and willingness to enforce "the rule of law." On the one hand, we can think of private corruption that a strong government may want to combat. This introduces an alternative form to use collected resources in a productive way. An interesting question is then what the optimal degree of corruption depends on if a given amount of tax revenues must be allocated towards competing productive tasks. This goes beyond Mauro (1996) who introduces corruption as a proportional tax on income in the setup of Barro (1990) and finds no distortion in the composition of public spending. On the other hand, the government itself may be weak and corrupt, hence, an impediment to economic growth.[28] One way to incorporate the consequences of inefficient government behavior is to assume that the government cannot transform collected tax revenues one-to-one into, say, productive public infrastructure. Finally, an interesting and related question concerns the determinants of the share of productive government expenditure. While in the models discussed above θ_G was either exogenous or chosen optimally by a planner, in reality this parameter reflects fundamental characteristics of the process of collective decision-making and the distribution of preferences and endowments (see, e. g., Alesina and Rodrik (1994)).

How about the role of productive government expenditure for sustained economic growth once we leave the well-defined analytical framework based on Barro (1990)? Arguably, one weakness of this approach is the knife-edge assumption of constant returns to scale (see, e. g., Solow (1994) for a critique of such assumptions). We have seen in Section 5.2 that increasing returns substantially alter the predictions of the growth performance. While the presence of increasing returns is empirically not implausible the policy recommendations of these models are hard to formulate since there is no natural way to select among multiple equilibria. Clearly, more research is needed here.

[28] See Acemoglu (2005) for a different notion of weak and strong states and their implications for economic development.

Some authors argue forcefully against the framework of Barro (1990) because neither the prediction of scale effects nor the dependency of the steady-state growth rate on taxation finds empirical support (see, e. g., Peretto (2003)). Indeed, the steady-state growth rate generated by non-scale models tends to be independent of government activity and the size of the economy. However, as we have seen in Section 5.3, the steady-state growth rate in the model of Eicher and Turnovsky (2000) is entirely determined by the technology of the economy and its consistency with a balanced-growth path. The role of economic agents is then quite passive. Moreover, in cross-country growth regressions the partial correlation between population growth and the growth rate of per-capita GDP is often found to be negative (see, e. g., Barro and Sala-í-Martin (2004) or Kormendi and Meguire (1985)).

In any case, it seems fair to say that the main body of the existing literature on productive government expenditure and economic growth is rooted in the tradition of investment-based endogenous growth models. In view of the strength and weaknesses of this approach it will be desirable in future research to incorporate productive government expenditure into idea-based endogenous growth models. This allows to address new questions, e. g. related to the effect of government activity on the productivity of an economy's research technology. On the other hand, these studies will generate findings that should be compared to those presented in this paper in order to select robust policy implications.

7 Appendix: The Pareto-Efficient Allocation of Section 4.1

Derivation of Equation (4.7)

The present-value Hamiltonian for the social planner's optimization problem is

$$H = lnc\, e^{-\rho t} + \lambda e^{-\rho t}\left[(1-\theta_G)AN^{\alpha(\sigma_G-1)}G^{\alpha}k^{1-\alpha} - c\right] + \upsilon e^{-\rho t}\theta_G AN^{\alpha(\sigma_G-1)+1}G^{\alpha}k^{1-\alpha}.$$

The optimality conditions with respect to c, k and G, for given θ_G, then obtain as

$$\frac{1}{c} = \lambda \tag{7.1}$$

$$(1-\alpha)AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha}[(1-\theta_G)+\mu N\theta_G] = \rho - \frac{\dot{\lambda}}{\lambda} \tag{7.2}$$

$$\frac{\alpha AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha-1}}{\mu}[(1-\theta_G)+\mu N\theta_G] + \frac{\dot{\mu}}{\mu} = \rho - \frac{\dot{\lambda}}{\lambda}, \tag{7.3}$$

where $\mu \equiv \upsilon/\lambda$ denotes the endogenously determined shadow value of public capital in terms of private capital.

Then, (7.1) to (7.3) deliver the planner's consumption growth rate, γ_c^P, and a differential equation describing the evolution μ

$$\gamma_c^P = (1-\alpha)AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha}[(1-\theta_G)+\mu N\theta_G] - \rho, \tag{7.4}$$

$$\dot{\mu} = \left[\frac{(1-\alpha)\mu G}{k} - \alpha\right]AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha-1}[(1-\theta_G)+\mu N\theta_G]. \tag{7.5}$$

The growth rates of private and public capital are given by (4.4) and (4.5). For convenience, we repeat them here

$$\gamma_G = \theta_G AN^{\alpha(\sigma_G-1)+1}\left(\frac{G}{k}\right)^{\alpha-1}, \tag{7.6}$$

$$\gamma_k = (1-\theta_G)\,AN^{\alpha(\sigma_G-1)}\left(\frac{G}{k}\right)^{\alpha} - \frac{c}{k}. \tag{7.7}$$

As the steady-state equilibrium of this economy is one in which consumption, private and public capital all grow at the same rate, it is convenient to express equations (7.4)-(7.7) in terms of the stationary variables $z \equiv G/k$ and $x \equiv c/k$. Then, the following set of differential equations determines the equilibrium dynamics of this economy

$$\frac{\dot{z}}{z} = \theta_G AN^{\alpha(\sigma_G-1)+1}z^{\alpha-1} - (1-\theta_G)AN^{\alpha(\sigma_G-1)}z^{\alpha} + x \tag{7.8}$$

$$\frac{\dot{x}}{x} = [(1-\theta_G)+\mu N\theta_G](1-\alpha)AN^{\alpha(\sigma_G-1)}z^{\alpha} - \rho - (1-\theta_G)AN^{\alpha(\sigma_G-1)}z^{\alpha} + x \tag{7.9}$$

$$\dot{\mu} = [(1-\alpha)\mu z - \alpha]AN^{\alpha(\sigma_G-1)}z^{\alpha-1}[(1-\theta_G)+\mu N\theta_G]. \tag{7.10}$$

Further, the following transversality conditions must hold

$$\lim_{t\to\infty}\lambda k e^{-\rho t} = 0 \quad \text{and} \quad \lim_{t\to\infty}\upsilon G e^{-\rho t} = 0.$$

The steady-state condition $\dot{z} = \dot{x} = \dot{q} = 0$ delivers

$$x - (1-\theta_G)AN^{\alpha(\sigma_G-1)}z^{\alpha} = -\theta_G AN^{\alpha(\sigma_G-1)+1}z^{\alpha-1}, \tag{7.11}$$

$$x - (1-\theta_G)AN^{\alpha(\sigma_G-1)}z^{\alpha} = \rho - [(1-\theta_G) + \mu N\theta_G](1-\alpha)AN^{\alpha(\sigma_G-1)}z^{\alpha}, \tag{7.12}$$

$$[(1-\alpha)\mu z - \alpha]\, AN^{\alpha(\sigma_G-1)}z^{\alpha-1}\,[(1-\theta_G) + \mu N\theta_G] = 0. \tag{7.13}$$

Equation (7.13) implies that the steady-state value of μ is given by

$$\mu = \frac{\alpha}{1-\alpha}\frac{1}{z}. \tag{7.14}$$

Substituting (7.14) into (7.4) delivers

$$\begin{aligned}
\gamma_c^P &= AN^{\alpha(\sigma_G-1)}z^{\alpha-1}\,[(1-\theta_G)(1-\alpha)z + \alpha N\theta_G] - \rho \\
&= (1-\theta_G)\underbrace{(1-\alpha)AN^{\alpha(\sigma_G-1)}z^{\alpha}}_{\partial y/\partial k} + \theta_G N\underbrace{\alpha AN^{\alpha(\sigma_G-1)}z^{\alpha-1}}_{\partial y/\partial G} - \rho,
\end{aligned} \tag{7.15}$$

such that (7.15) corresponds to equation (4.7) of Section 4.1 with y given by (4.3).

Derivation of θ_G^P of Equation (4.11)

Maximizing the Hamiltonian with respect to θ_G, i. e., $\partial H/\partial\theta_G = 0$, delivers

$$\begin{aligned}
\lambda AN^{\alpha(\sigma_G-1)}G^{\alpha}k^{1-\alpha} &= \upsilon AN^{\alpha(\sigma_G-1)+1}G^{\alpha}k^{1-\alpha} \\
\mu^P &= \frac{1}{N}.
\end{aligned}$$

Hence, for an unconstrained Pareto-optimum $\dot{\mu} = 0$ is required. From (7.14) it follows that $z^P = \alpha N/(1-\alpha)$, and thus $\dot{z} = 0$. Then, from (7.9) and the transversality condition we know that also x must be constant at all times. Equalizing the right-hand sides of (7.11) and (7.12) and substituting z^P and μ^P gives

$$\theta_G AN^{\alpha\sigma_G}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} = (1-\alpha)AN^{\alpha\sigma_G}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} - \rho \tag{7.16}$$

$$\frac{\rho\alpha}{1-\alpha} = (\alpha-\theta_G)AN^{\alpha\sigma_G}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}. \tag{7.17}$$

Resubstituting (7.17) into (7.12) we obtain

$$\begin{aligned}
x^P &= \rho + (\alpha-\theta_G)AN^{\alpha\sigma_G}\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \\
&= \rho + \frac{\rho\alpha}{1-\alpha} \\
&= \frac{\rho}{1-\alpha}.
\end{aligned}$$

Moreover, solving (7.16) for θ_G delivers

$$\theta_G^P = \alpha - \frac{\rho}{AN^{\alpha\sigma_G}\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}}, \tag{7.18}$$

which corresponds to (4.11) in the main text.

Derivation of τ_y^P of Equation (4.12)

From (7.4) with $\mu = \mu^P$ we obtain

$$\gamma_c^P = (1-\alpha)AN^{\alpha(\sigma_G-1)}\left(z^P\right)^{\alpha} - \rho. \tag{7.19}$$

Then, comparing γ_c of (4.2) to γ_c^P of (7.19) reveals that

$$\gamma_c = \gamma_c^P \quad\Leftrightarrow\quad (1-\tau_y)(1-\sigma_G\,\alpha) = (1-\alpha).$$

Thus, the income tax rate that implements the Pareto-efficient allocation is given by

$$\tau_y^P = \frac{\alpha\,(1-\sigma_G)}{1-\sigma_G\,\alpha},$$

which corresponds to (4.12).

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